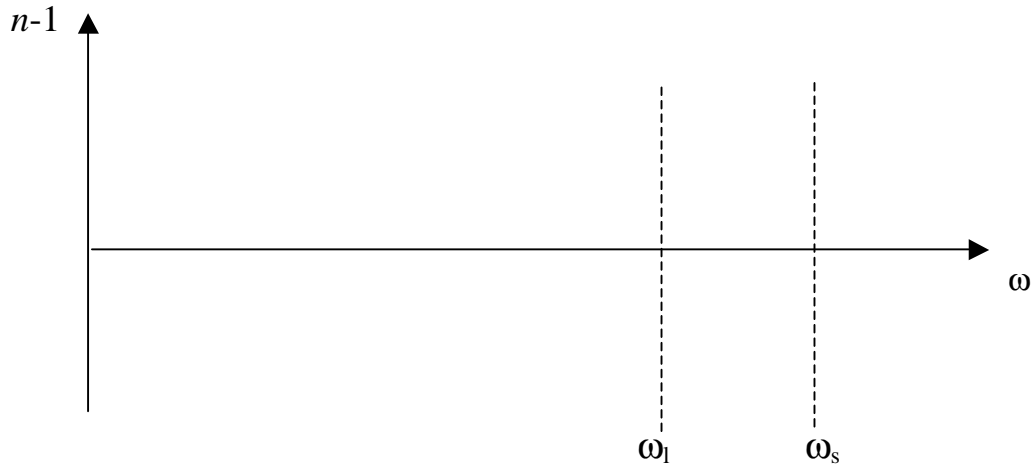


Rules: open books, notes, internet. NO communication with other people. Neat, clear work.
 Begin each problem on a new page.

1. Suppose we have a gas of a molecule that has a long axis and **two, equal** short axes. If we apply a static electric field along the x-direction, the molecules will align with the long axis in that direction. We will use a simple, single-resonance model for the response of the molecule to an EM wave. Along the short and long directions of the molecule, the resonance frequencies of the bound electrons are ω_s and ω_l respectively. Assume the damping coefficient is the same for both resonances, and that the oscillator strengths (f_α) are equal to 1. Since the electrons are less tightly bound along the long direction, $\omega_l < \omega_s$.
 - a. Consider a plane wave propagating in the z-direction through this gas of molecules aligned in the x-direction. Write two expressions for the complex refractive index (n_x and n_y) experienced by light polarized in the x- and the y-directions. It is not necessary to derive these expressions.
 - b. Sketch two curves for the real part of the refractive index that corresponds to these two resonance frequencies.



- c. Suppose we measure the refractive indices at a wavelength of 500nm and we find that $n_x-1 = 5 \times 10^{-4}$ at standard atmospheric pressure, and n_y-1 is 20% **lower**. The gas is transparent at this wavelength. Estimate values for the two resonance frequencies.
- d. A plane wave of wavelength $\lambda=500\text{nm}$, with linear polarization oriented at 45° to the x-axis is directed into a gas cell that is 10 cm long. **The gas cell has the molecules aligned as described above, and this alignment is unaffected by the input wave.** The pressure is brought up from vacuum to a point where the output polarization is observed to rotate by 90° . What pressure is required for this to occur? Be sure to show how you arrive at your answer.

2. Consider a metallic rectangular waveguide filled with a dielectric with an index of refraction n , and with widths a and b along the x and y directions, respectively. The propagation direction is in z (see Fig. 7-5 in Heald and Marion).
- For the TM modes, write an expression for the longitudinal component of the electric field for the guided modes, $E_z(x, y, z, t)$. Describe the boundary conditions you are applying for this field component. Define the values of the x , y and z components of the k -vector.
 - Show that the lowest-order mode is TM_{11} .
 - Now assume the dielectric inside the waveguide is a plasma. Calculate an expression for the z -component for the k -vector, k_z expressed in terms of the number density N_e of the free electrons in the plasma.
 - Calculate an expression for the cutoff frequencies ω_{mn} for this plasma-filled waveguide, where m, n are the mode indices for the x and y directions. Show your work and explain the basis for your reasoning.
 - Suppose the waveguide is square, with $a = b = 0.01$ mm, and the waveguide is empty (vacuum). What is the cutoff wavelength for the lowest mode?
 - If the input wavelength to the square waveguide from part e is $7\mu\text{m}$, how many modes does the empty waveguide support? At what number density for the plasma will the waveguide only support a single mode?

3. Consider the radiation damping of a moving, oscillating charge. An electron is launched **at the origin** with a velocity components v_{x0}, v_{z0} into a region where there is a potential

$$U(x, z) = \frac{1}{2} m \omega_0^2 x^2.$$

- Calculate the initial time average radiated power, assuming all velocities are much less than c .
- Describe how radiation damping affects the motion of the electron: make sketches of the $x(t)$ and $z(t)$. Explain your reasoning.
- Sketch the angular distribution of the radiated power in the low velocity limit $v \ll c$, and in the limit where v_{z0}/c is appreciable. Support your answer by relating this physical situation to other examples in the book.