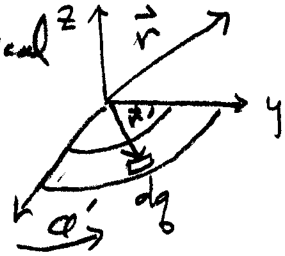
3. A uniform surface charge density, σ is placed on a quarter of a ring with inner radius R_a and outer radius R_b as shown on the chalk board. Derive an expression for the electric field at an arbitrary position.

Fund. Prin. $\vec{E} = \int \frac{k\,dq}{\eta^2}\hat{\eta}$ choose cylindrical coords



$dq = \sigma s' d\varphi' ds'$

$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

$\vec{r}\,' = s'\cos\varphi'\hat{x} + s'\sin\varphi'\hat{y}$

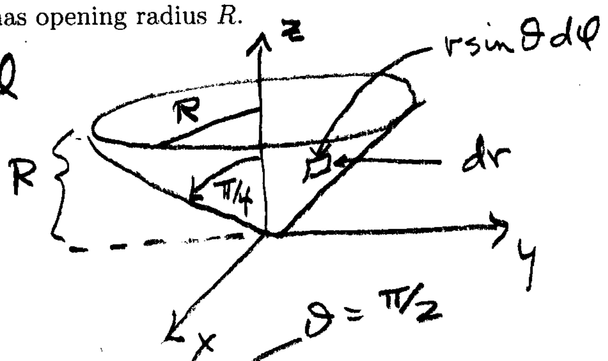
$\vec{\eta} = \vec{r} - \vec{r}\,' = (x - s'\cos\varphi')\hat{x} + (y - s'\sin\varphi')\hat{y} + z\hat{z}$

$$\vec{E} = \int_{R_a}^{R_b}\int_0^{\pi/2} \frac{1}{4\pi\epsilon_0}\frac{\sigma s' d\varphi' ds'}{\eta^3}\vec{\eta}$$

$$|\vec{\eta}| = \left[(x - s'\cos\varphi')^2 + (y - s'\sin\varphi')^2 + z^2\right]^{1/2}$$

4. Derive an expression for the flux of the electric field $\vec{E} = 3\hat{r} + 4r^3\hat{\theta} + 5\hat{\phi}$ through the surface of a cone which makes an angle of $\pi/4$ relative to the z axis and has opening radius R.

Fund. Prin. $\Phi = \int \vec{E}\cdot d\vec{a}$ spherical coords



$d\vec{a} = \pm r\sin\theta\, d\varphi\, dr\,\hat{\theta}$

↑ normal could point in 2 directions

$\theta = \pi/2$

$$\Phi = \int \vec{E}\cdot d\vec{a} = \pm\int (3\hat{r} + 4r^3\hat{\theta} + 5\hat{\varphi})\cdot r\sin\theta\, d\varphi\, dr\,\hat{\theta}$$

$$= \pm\int_0^{\sqrt{2}R}\int_0^{2\pi} 4r^3 r\sin\theta\, d\varphi\, dr$$

r goes from 0 to hypotenuse