3a) Ok, so $V(x,y,z)$ must be zero for <u>all</u> y and z when $x=0$ or $x=W$. If that has to be true for all y and z, it must be something we accomplish purely with the x term, so:

$$\sin(K_{1n}\cdot 0) = \sin(K_{1n}W) = 0$$

The first of those isn't helpful - $\sin(0)=0$ no matter what.

But the second is helpful. it tells us $K_{1n}W$ must equal some integer multiple of π:

$$K_{1n}W = n\pi \Rightarrow \boxed{K_{1n} = \frac{n\pi}{W}}$$

Identical arguments in the y direction yield $\boxed{K_{2m} = \frac{m\pi}{L}}$

Which means $\boxed{K_{3mn} = \sqrt{\left(\frac{n\pi}{W}\right)^2 + \left(\frac{m\pi}{L}\right)^2}}$

b) Our last condition is that $V=V_0$ at $z=0$ for all x and y. At $z=0$, $e^{-K_{3mn}z}=1$, so we get:

$$V(x,y,0) = V_0 = \sum_{n=1}\sum_{m=1} C_{mn}\sin\left(\frac{n\pi x}{W}\right)\sin\left(\frac{m\pi y}{L}\right)$$

We can extract the C_{mn} using Fourier's trick in 2D. Multiply both sides by $\sin\left(\frac{l\pi x}{W}\right)\sin\left(\frac{j\pi y}{L}\right)$ and integrate to exploit orthogonality:

$$\int_0^L\int_0^W V_0\sin\left(\frac{l\pi x}{W}\right)\sin\left(\frac{j\pi y}{L}\right)dx\,dy =$$

$$\sum_n\sum_m \int_0^L\int_0^W \sin\left(\frac{l\pi x}{W}\right)\sin\left(\frac{n\pi x}{W}\right)\sin\left(\frac{j\pi y}{L}\right)\sin\left(\frac{m\pi y}{L}\right)dx\,dy$$

And we know from math phys (take a drink):

$$\int_0^W \sin\left(\frac{l\pi x}{W}\right)\sin\left(\frac{n\pi x}{W}\right)dx = \delta_{ln}\frac{W}{2}$$

$$\int_0^L \sin\left(\frac{j\pi y}{L}\right)\sin\left(\frac{m\pi y}{L}\right)dy = \delta_{jm}\frac{L}{2}$$

The Kronecker deltas collapse the sums, giving us

$$V_0\int_0^L \sin\left(\frac{j\pi y}{L}\right)dy\int_0^W \sin\left(\frac{l\pi x}{W}\right)dx = C_{lj}\cdot\frac{W}{2}\frac{L}{2}$$

And $$\int_0^L \sin\left(\frac{j\pi y}{L}\right)dy = -\frac{L}{\pi j}\cos\left(\frac{j\pi y}{L}\right)\Big|_0^L$$

$$= \frac{-L}{\pi j}\left[\cos(j\pi) - \cos(0)\right]$$

Here's a tricky part: When $j=0$, $\cos(j\pi)=1$ and the integral yields 0. But when $j=1$, $\cos(j\pi)=-1$ and the integral gives not-zero. This pattern continues. For even j we get zero, for odd we get

$$= -\frac{L}{\pi j}\left[-1-1\right] = \frac{2L}{\pi j}.$$

Therefore $$\int_0^W \sin\left(\frac{l\pi x}{W}\right)dx = \frac{2W}{\pi l}$$ if l is odd, 0 if even

And so

$$V_0\cdot\frac{2W}{\pi l}\cdot\frac{2L}{\pi j} = \frac{W}{2}\frac{L}{2}C_{lj} \Rightarrow C_{lj} = \frac{16V_0}{\pi^2 lj}$$ if l and j are both odd, zero otherwise

c) Putting it all together and relabeling indices:

$$\boxed{V(x,y,z) = \sum_{n=1,3,5\ldots}^{\infty}\sum_{m=1,3,5\ldots}^{\infty}\frac{16V_0}{\pi^2 nm}e^{-\sqrt{\left(\frac{n\pi}{W}\right)^2+\left(\frac{m\pi}{L}\right)^2}\,z}\sin\left(\frac{n\pi x}{W}\right)\sin\left(\frac{m\pi y}{L}\right)}$$

That is beefy.