Electrostatics

+

$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0 \qquad \vec{E} = -\vec{\nabla} V$$

$$= -\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \cdots\right) V$$

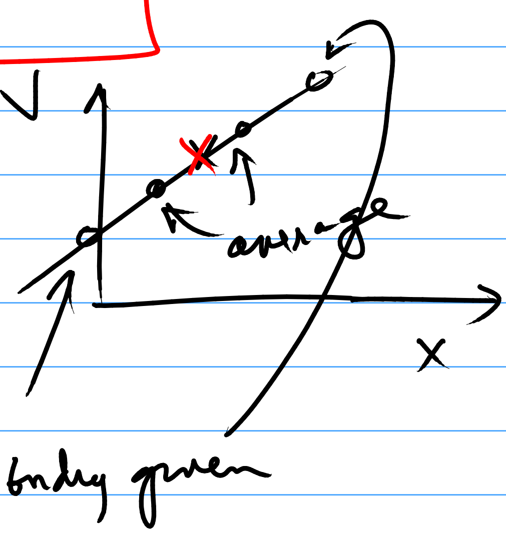
$$\downarrow$$

$$\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) \cdot \left(-\hat{x}\frac{\partial V}{\partial x} - \hat{y}\frac{\partial V}{\partial y} - \hat{z}\frac{\partial V}{\partial z}\right) = \frac{\rho}{\epsilon_0}$$

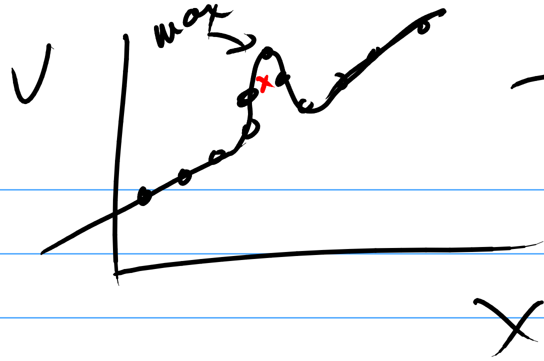
$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\rho/\epsilon_0 \qquad \text{Poisson's eq}$$

IN VAC $\quad \boxed{\nabla^2 V = 0}$

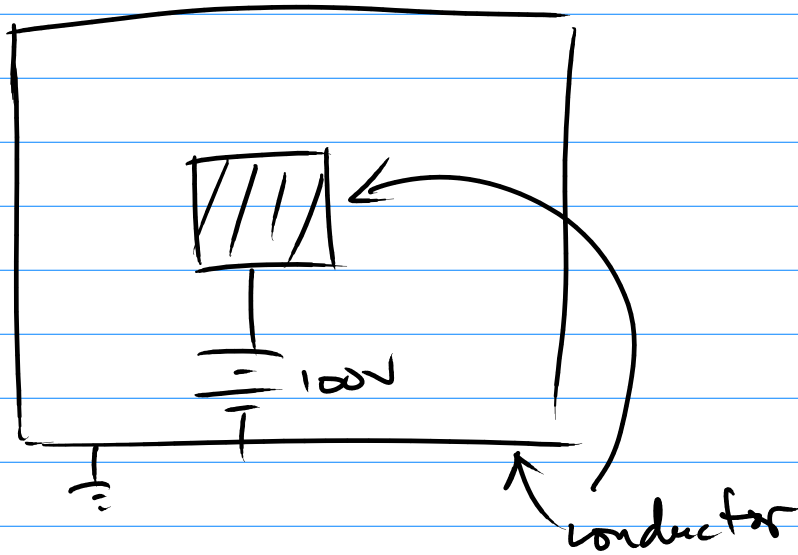
1-D $\quad \dfrac{\partial^2 V}{\partial x^2} = 0$

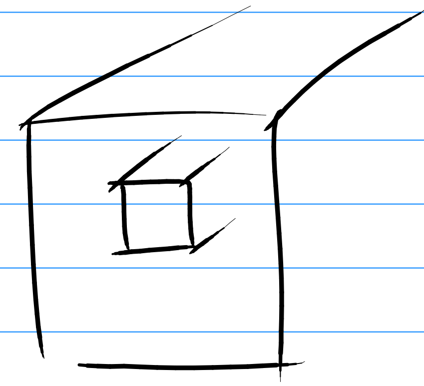


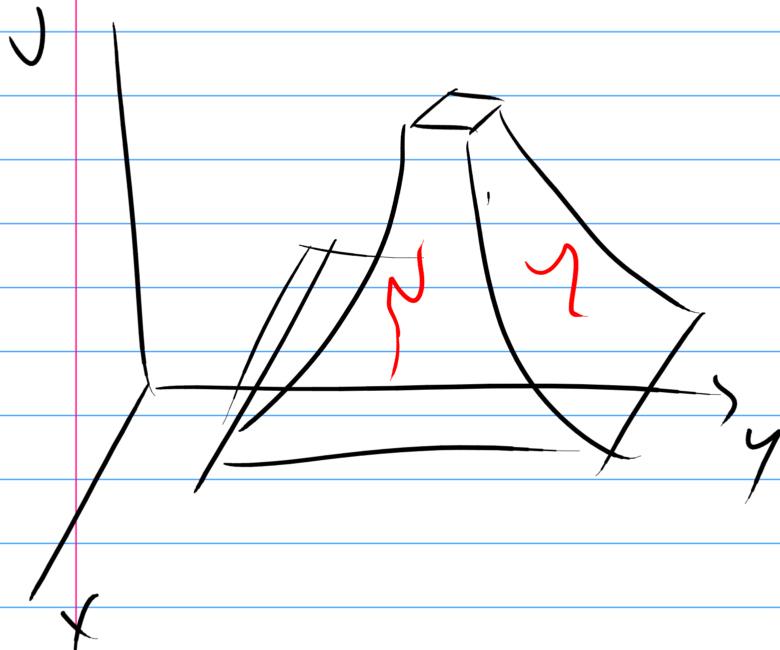
1. V can have no max or min anywhere except at bndry



Does not happen





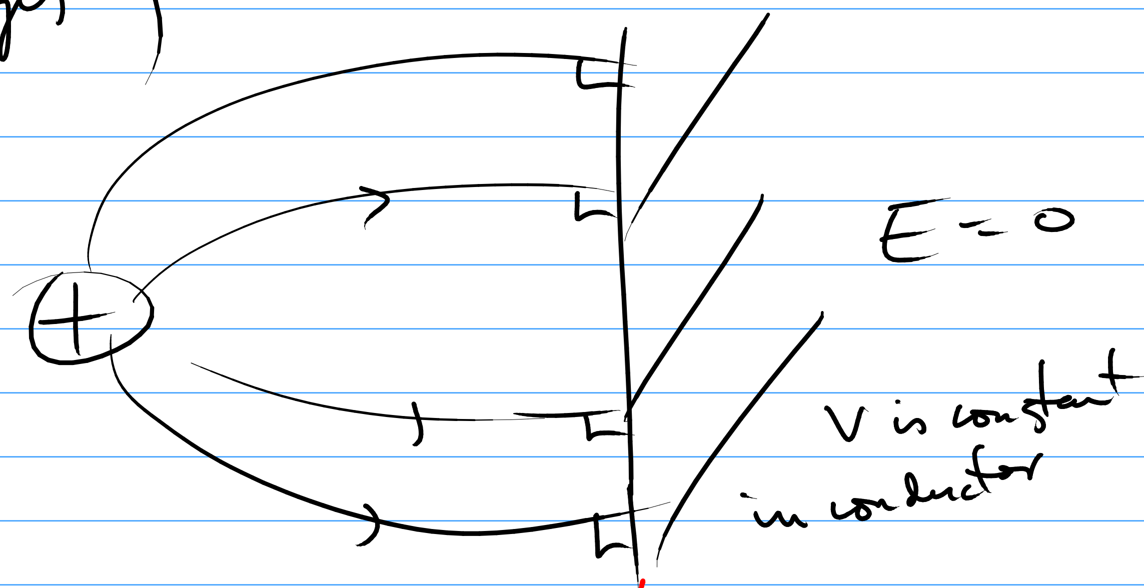


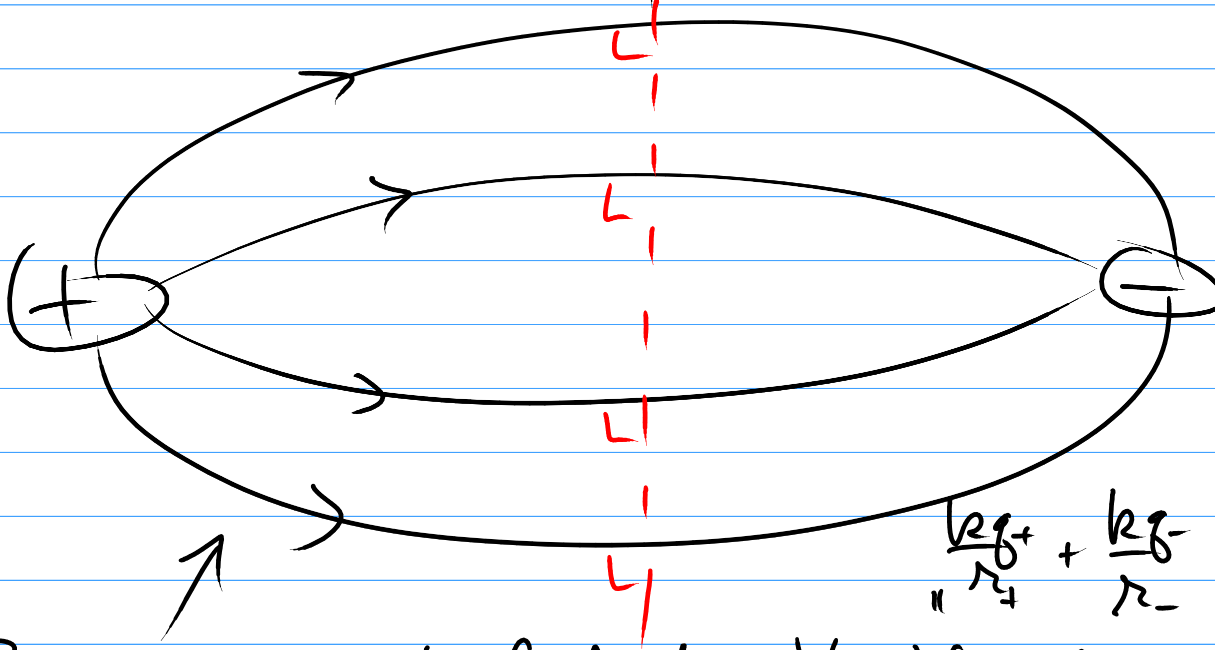
- Relaxation method of solving PDE

$\nabla^2 V = 0$

- Guess: both solve PDE & satisfy bndry condition

(method of images)

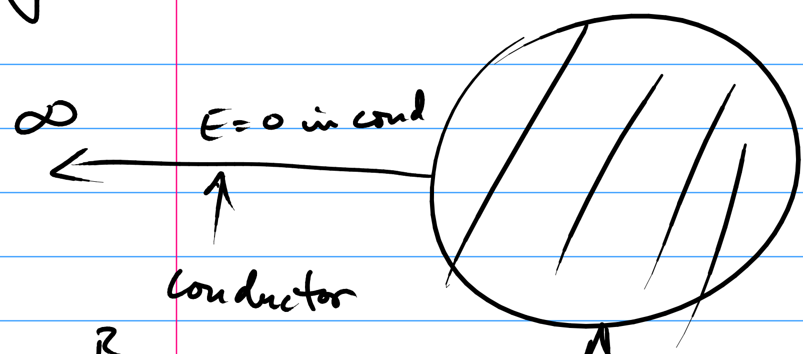




$\nabla^2 V = 0$ is satisfied by $V = V_{\oplus} + V_{\ominus}$

bndy cond is satisfied

$V = 0$ at ∞

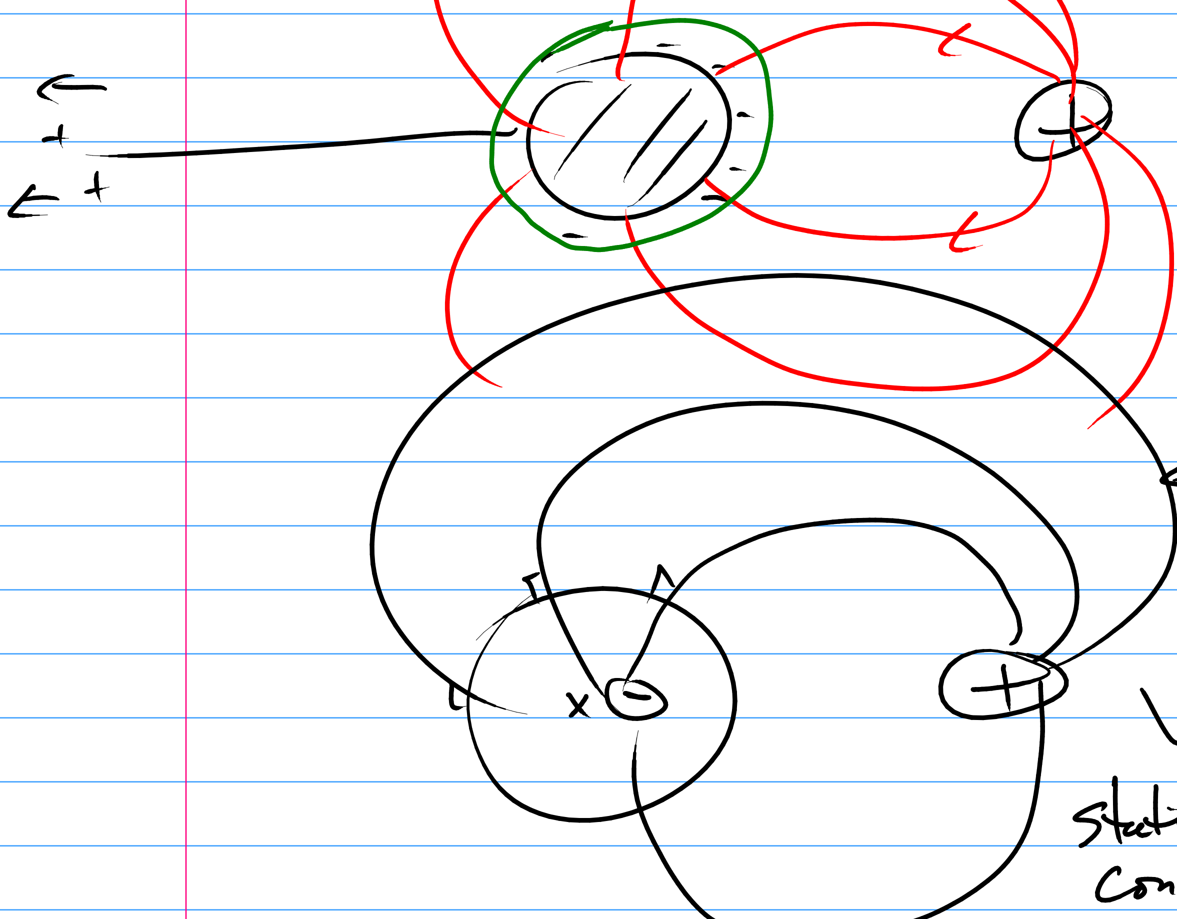


$$\int_{\infty}^{R} \vec{E}\cdot d\vec{l} = 0$$

$V = 0$

neg

$$\oint \vec{E}\cdot da = \frac{Q_{encl}}{\epsilon_0}$$



E & V satisfy PDE

$V = \frac{kq}{r}$

satisfy bndy cond.