MATH 225 - May 21, 2008
Quiz II - 25 minutes

NAME: Key

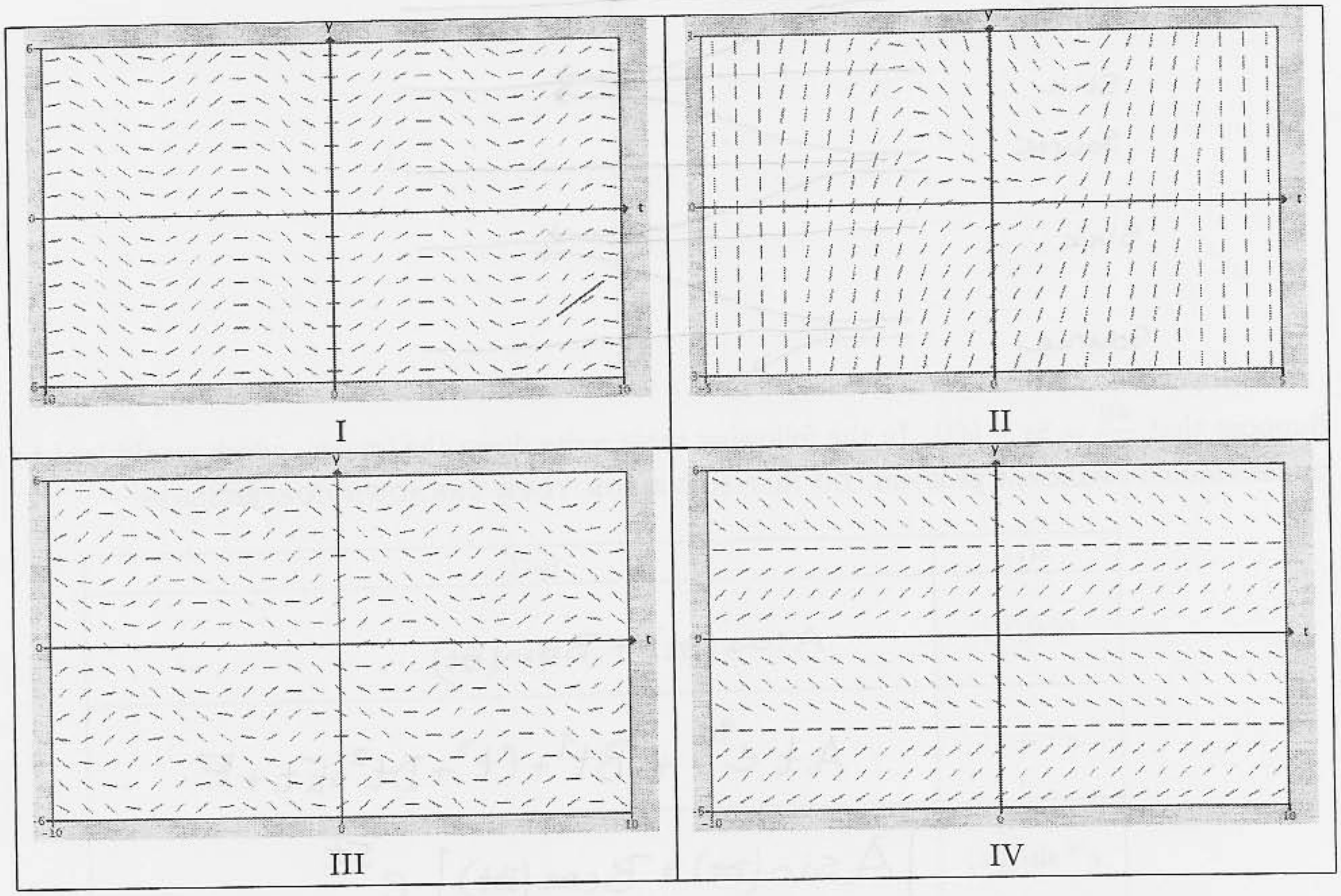
THERE ARE TWO SIDES TO THIS QUIZ

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

1. Match the following differential equations to their slope fields.

(a) $\dfrac{dy}{dt} = t^2 - y$ II (b) $\dfrac{dy}{dt} = \sin(y)$ IV

(c) $\dfrac{dy}{dt} = \sin(t)$ I (d) $\dfrac{dy}{dt} = \sin(t+y)$ III



2. Find the solution to the IVP, $y' + y = \sin(t)$, $y(0) = -1$.

$y_h(t) = e^{-t}$, $b(t) = \sin(t) \Rightarrow y_p(t) = A\sin(t) + B\cos(t)$

$\Rightarrow y' + y = A\cos(t) + -B\sin(t) + A\sin(t) + B\cos(t) = \sin(t)$

$\Rightarrow A + B = 0$, $A - B = 1 \Rightarrow A = \frac{1}{2}$, $B = -\frac{1}{2}$

Hence,

$Y(t) = Ke^{-t} + \frac{1}{2}\sin(t) + \frac{-1}{2}\cos(t)$. $Y(0) = -1 = K + \frac{-1}{2}\cos(0)$

$K = -\frac{1}{2}$

$Y(t) = -\frac{1}{2}e^{-t} + \frac{-1}{2}\cos(t) + \frac{1}{2}\sin(t)$

3. Given the first-order autonomous ODE,

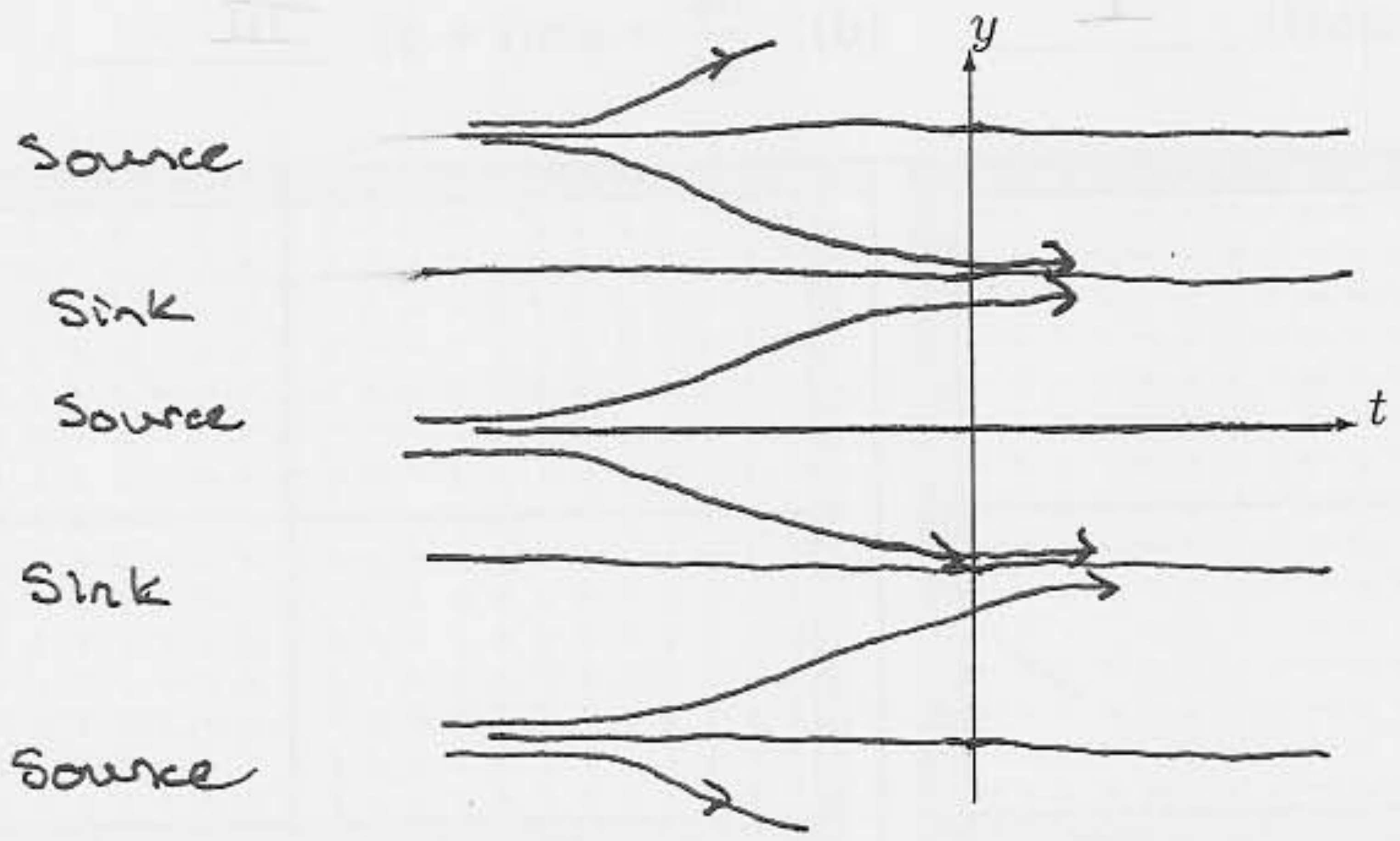
$$\frac{dy}{dt} = \sin(y) \qquad (1)$$

(a) Find and classify all equilibrium solutions to the ODE.

$\frac{dy}{dt} = 0 = \sin(y) \Rightarrow y_n = n\pi, \quad n = 0, \pm 1, \cdots$

$f'(y) = \cos(y) \Rightarrow f'(y_n) = \cos(n\pi) = \begin{cases} 1, & n \equiv \text{Even or zero} \\ -1, & n \equiv \text{odd} \end{cases} \Rightarrow \begin{matrix} y_n \equiv \text{source} \\ y_n = \text{sink} \end{matrix}$

(b) Using a phase line analysis draw possible solution trajectories in the ty-plane for $y \in [-2\pi, 2\pi]$.



4. Suppose that $\frac{dy}{dt} = 5y + b(t)$. In the following table write down the 'guess', which would lead to a solvable undetermined coefficient problem. DO NOT SOLVE FOR YOUR UNKNOWN COEFFICIENTS

$b(t)$	$y_p(t)$
$\cos(\beta t)$	$A\cos(\beta t) + B\sin(\beta t)$
$e^{5t} + t^4$	$Ate^{5t} + Bt^4 + Ct^3 + Dt^2 + Et + F$
$e^{5t}\sin(\beta t)$	$[A\sin(\beta t) + B\cos(\beta t)]\, e^{5t}$
$t^2 e^t \cos(\beta t)$	$(At^2 + Bt + \cancel{D}C)e^t\,(D\cos(\beta t) + E\sin(\beta t))$

5. Given, $\frac{dy}{dt} = f(t, y)$. If f is discontinuous at $(0,0)$ but $\frac{\partial f}{\partial y}$ is not then do solutions to the ODE passing through $(0,0)$ exist? Are these solutions unique? Explain.

It is unknown if Soln Exist at $(0,0)$ if one does Exist then it is unique since $\frac{\partial f}{\partial y}$ is Continuous at $(0,0)$