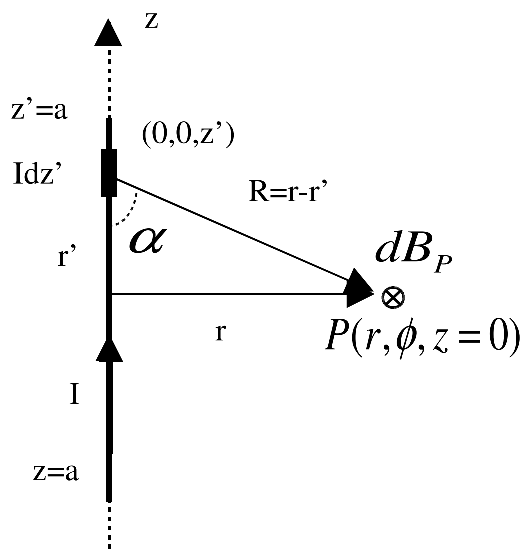
PROBLEM SOLVING STRATEGY STEPS

Problem 1: Consider a straight, current carrying wire of length 2a. Find the B field at a point $P(r,\phi,0)$ equidistant from the end points of the conductor.

Solution

I STEP: IDENTIFY THE FUNDAMENTAL PRINCIPLES

- **Construct a mental image of the sequence of events described in the problem statement.**
- **Sketch a picture which represents this mental image, include the given information**
- **Make sure all symbols representing quantities shown on picture or diagram are defined.**
- **Determine what the question is asking.**
- **Construct a model to simplify the calculation if necessary.**
- **Select a qualitative approach which should lead to a solution to the problem**



II. STEP: OUTLINE THE SOLUTION

- **State quantitative relationships from general principles and specific constraints**
- **Declare a target quantity**
- **Solve for the target quantity**
- **Put in the numerical value and units for each quantity in your equation for the target quantity**
- **Check that each additive term of the solution has the same type of units**
- **Calculate the numerical value of the target quantity by combining the numbers with arithmetic and units with algebra**

Since we have azimuthally symmetry (i.e., no dependence on ϕ), a cylindrical coordinate with the wire oriented along the z-axis and centered at the origin is the appropriate. The field dB_P at point P (note that $r = \hat{r}r$) due to current element $IdI^{'}$, located at $r' = \hat{z}z'$ at a distance $R = |r - r'| = \sqrt{r^2 + (z')^2}$ from point P is

$$\vec{dB_P} = \frac{\mu_0}{4\pi}\frac{Idz'\, zx\hat{R}}{R^2} = \hat{\Phi}\frac{\mu_0 I}{4\pi}\frac{\sin\alpha\, dz'}{\left[\left(r^2 + (z')^2\right)\right]}$$

$$= \hat{\Phi}\frac{\mu_0 I}{4\pi}\frac{rdz'}{\left[\left(r^2 + (z')^2\right)\right]^{3/2}}$$

Where we have used the fact that the magnitude of the cross product of two vectors is equal to the product of the magnitude of two magnitudes of the vectors times the sin of the angle between them; that is, $\hat{z}x\hat{R} = |\hat{z}||\hat{R}|\sin\alpha = \hat{\Phi}\sin\alpha$ Note that $\sin\alpha = r\left[\left(r^2 + (z')^2\right)\right]^{-1/2}$ with α as shown in Figure. We can find the total B_P field at point P by integrating dB_P over the length of the wire,

$$\vec{B_P} = \hat{\Phi}\frac{\mu_0 Ir}{4\pi}\int_{z'=-a}^{+a}\frac{dz'}{\left[r^2 + (z')^2\right]^{3/2}}$$

$$= \hat{\Phi}\frac{\mu_0 Ir}{4\pi}\left[\frac{z'}{r^2\sqrt{r^2 + (z')^2}}\right]_{z'=-a}^{+a}$$

$$= \hat{\Phi}\frac{\mu_0 Ia}{2\pi r\sqrt{r^2 + a^2}}$$

III. STEP: CHECK THE ANSWER

- **Check that answer is properly stated.**
- **Convert units as necessary to simplify the expression for the target quantity in terms of an understandable set of units and answer the question.**
- **Check that answer is reasonable.**
- **Review the problem solution.**
- **Determine if the answer is complete.**
- **The solution to your model is the result.**

For an infinitely long conductor, or at small distances from a finite-length conductor (i.e., r<<a), we find

$$\vec{B}_P \cong \hat{\Phi}\frac{\mu_0 I}{2\pi r}$$

Problem 2: As a numerical example, what is the **B** field at a distance of 1 cm from a long straight wire carrying of I=5A?

Solution

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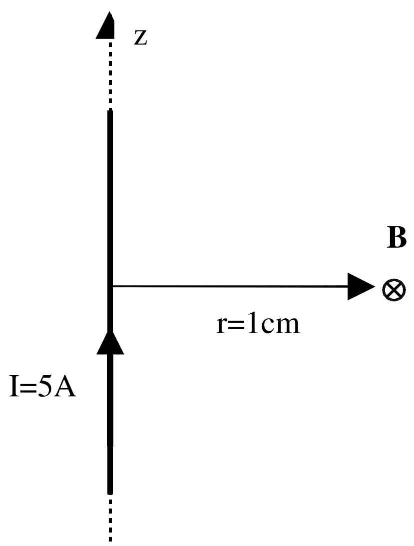

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$$B = \frac{\mu_0 I}{2\pi r}$$

$$\mu_0 = 4\pi x 10^{-7}\, tesla\, meter / ampere \;\; (permeability\; of\;\; free\; space)$$

$$B = ?$$
$$r = 1cm,\; I = 5A$$
$$r = 1cm = 0.01m$$

$$B = \frac{4\pi x 10^{-7}\,(T.m/A)\;5(A)}{2\pi 0.01(m)} = 1x10^{-4}T$$

III. STEP: CHECK THE ANSWER

- **Check that answer is properly stated**
- **Convert units as necessary to simplify the expression for the target quantity in terms of an understandable set of units and answer the question**
- **Check that answer is reasonable**
- **Review problem solution**

- **Determine if the answer is complete**
- **The solution to your model is the result.**

$$B = \frac{4\pi x10^{-7}(T.m/A)\ 5(A)}{2\pi 0.01(m)} = 1x10^{-4}T$$

$$B = 1x10^{-4}T = 1gauss$$

Problem 3: The space between the plates of a parallel-plate capacitor is filled with two slabs of linear dielectric material. Each slab has thickness a, so the total distance between the plates is 2a. Slab 1 has a dielectric constant of 2, and slab 2 has a dielectric constant of 1.5. The free charge density on the top plate is σ and on the bottom plate $-\sigma$.

a) Find the electric displacement **D** in each slab.
b) Find the electric field **E** is each slab.
c) Find the polarization **P** is each slab.

<u>**Solution**</u>

<u>**I STEP: IDENTIFY THE FUNDAMENTAL PRINCIPLES**</u>

- **Construct a mental image of the sequence of events described in the problem statement.**
- **Sketch a picture which represents this mental image, include given information**
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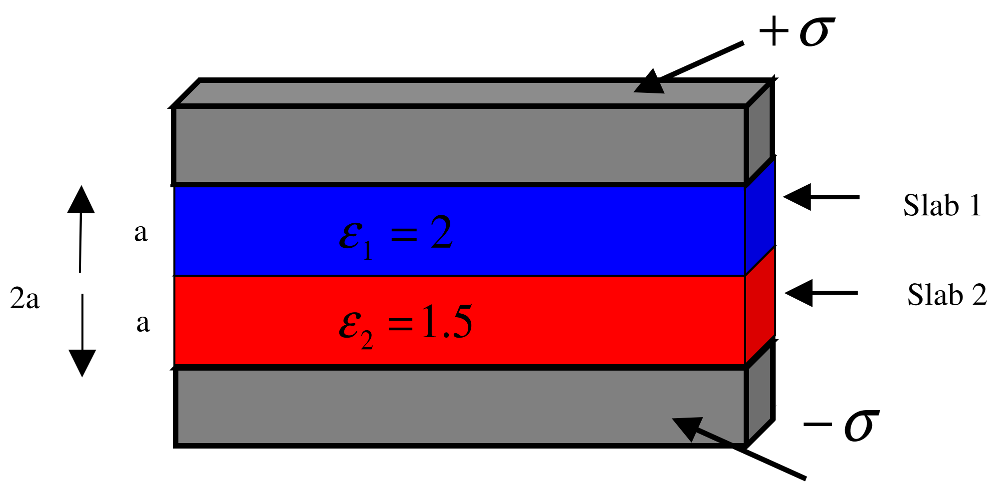

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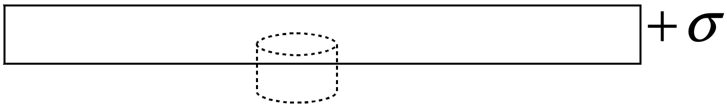
a) Apply $\int D.da = Q_{fenc}$ to the Gaussian surface shown

$$D.A = \sigma.A$$

$$D = \sigma$$

Note: **D**=0 inside the metal plate. This is true in both slabs; **D** points down



b)

$$D = \varepsilon E$$

$$E = \frac{\sigma}{\varepsilon_1} \; in\, slab 1$$

$$E = \frac{\sigma}{\varepsilon_2} in\, slab\, 2$$

$$But \;\; \varepsilon = \varepsilon_0 \varepsilon_r, \; so$$

$$\varepsilon_1 = 2\varepsilon_0 \;\; \rightarrow \;\; E_1 = \frac{\sigma}{2\varepsilon_0}$$

$$\varepsilon_2 = \frac{3}{2}\varepsilon_0 \rightarrow \;\; E_2 = \frac{2\sigma}{3\varepsilon_0}$$

c)

$$P = \varepsilon_0 \chi_e E$$

$$P = \varepsilon_0 \chi_e \frac{\sigma}{\varepsilon_0 \varepsilon_r} = \frac{\chi_e \sigma}{\varepsilon_r}$$

$$\varepsilon_r = 1 + \chi_e$$

$$P = \frac{(\varepsilon_R - 1)\sigma}{\varepsilon_R}$$

$$P = (1 - \varepsilon_r^{-1})\sigma \rightarrow P_1 = \frac{\sigma}{2}$$

$$P_2 = \frac{\sigma}{3}$$

III. STEP: CHECK THE ANSWER

- **Check that answer is properly stated**
- **Convert units as necessary to simplify the expression for the target quantity in terms of an understandable set of units and answer the question**
- **Check that answer is not unreasonable**
- **Review problem solution**
- **Determine if answer is complete**
- **The solution to your model is the result.**

a) $$D.A = \sigma.A$$ $$D = \sigma$$ This is true in both slabs; **D** points down

b) $E_1 = \dfrac{\sigma}{2\varepsilon_0}$; $E_2 = \dfrac{2\sigma}{3\varepsilon_0}$

c) $P_1 = \dfrac{\sigma}{2}$; $P_2 = \dfrac{\sigma}{3}$