

## *Comments from the grader*

1. They don't differential between vector and non-vector quantities. This is very annoying. Next time I will take off a lot more points for that.
2. Make sure that you include a printout of any work on Mathematica, or cite any references that you use for integrals.
3.  $\oint \mathbf{F} \cdot d\mathbf{z}$   
they don't understand that it must be a CLOSED SURFACE.
4. Problems with Gauss's and Ampere's Law.  
They don't understand the symmetry arguments; some state them even when they are NOT true.
5. Lack of diagrams and coordinate systems. Half the time, I had to guess their coordinates. Some had left handed coordinate systems (STRANGE!).

### Optical Element

### Jones Matrix

Linear Polarizer

$$\left\{ \begin{array}{l} \text{Transmission axis horizontal} \\ \text{" " vertical} \\ \text{" " at } \pm 45^\circ \end{array} \right. \quad \begin{array}{l} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\ \frac{1}{2} \begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix} \end{array}$$

$\frac{1}{4}$ -wave plate

$$\left\{ \begin{array}{l} \text{Fast axis vertical} \\ \text{" " horizontal} \\ \text{" " @ } \pm 45^\circ \end{array} \right. \quad \begin{array}{l} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \\ \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \pm i \\ \pm i & 1 \end{pmatrix} \end{array}$$

$\frac{1}{2}$ -wave plate

$$\text{Fast axis either horizontal or vertical} \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Isotropic phase retarder

$$\begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}$$

Relative phase changer

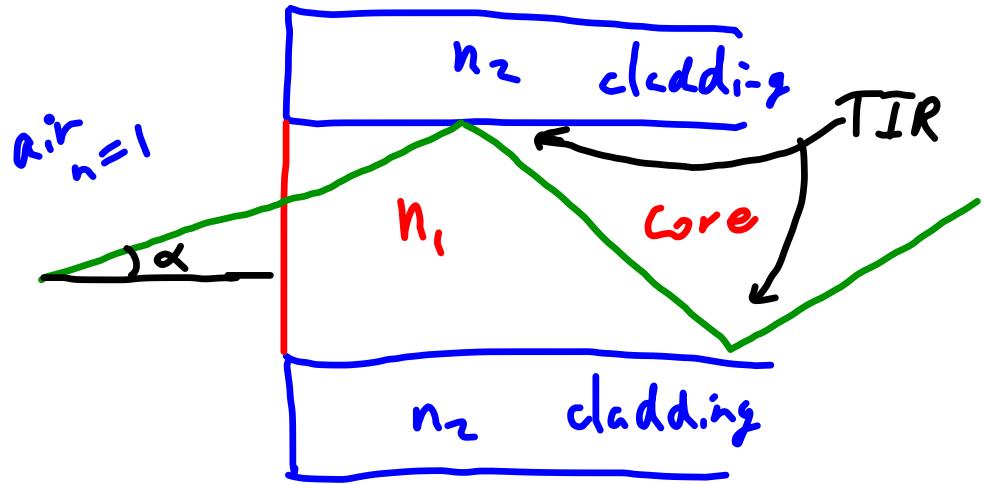
$$\begin{pmatrix} e^{i\phi_x} & 0 \\ 0 & e^{i\phi_y} \end{pmatrix}$$

Circular polarizer

$$\left\{ \begin{array}{l} \text{Right} \\ \text{Left} \end{array} \right. \quad \frac{1}{2} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \quad \frac{1}{2} \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$$

# Fiber optic / optical waveguide

$$n_1 > n_2$$



$\alpha$  = "acceptance angle"

$$\sin \alpha = \sqrt{n_1^2 - n_2^2}$$

Jones Calculus

$$\tilde{E} = \tilde{E}_0 e^{i(kz-\omega t)}$$

wave propagating along z direction  
transverse so  $E_z = 0$

$$\tilde{E}_0 = \tilde{E}_{0x} \hat{x} + \tilde{E}_{0y} \hat{y}$$

$$\begin{pmatrix} \tilde{E}_{0x} \\ \tilde{E}_{0y} \end{pmatrix} = \begin{pmatrix} E_{0x} e^{i\delta_x} \\ E_{0y} e^{i\delta_y} \end{pmatrix}$$

$$A + iB \\ r e^{i\phi} \\ \text{Im} \quad \text{Re}$$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$   
horizontal polarization

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$   
vertical polarization

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$   
left circular

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow e^{i(kz-\omega t)} [(\hat{x} + i\hat{y})] \\ \cos(kz-\omega t) + i \sin(kz-\omega t) [\hat{x} + i\hat{y}]$$

$$\cos(kz-\omega t) \hat{x} - \sin(kz-\omega t) \hat{y}$$

$\begin{pmatrix} 1 \\ -i \end{pmatrix}$  right circular

$\begin{pmatrix} 2 \\ i \end{pmatrix}$  left elliptical

$\begin{pmatrix} 1 \\ -2i \end{pmatrix}$  right elliptical

Add 2 waves to get a resultant wave, just add Jones vectors.

$$\begin{pmatrix} 1 \\ i \end{pmatrix} + \begin{pmatrix} 1 \\ -i \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

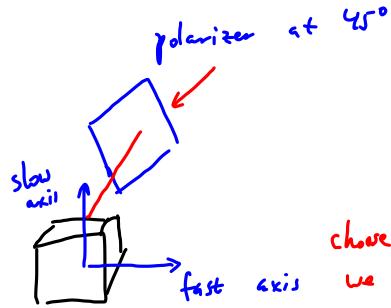
↓              ↓  
left circular    right circular

horizontal polarization  
with twice the amplitude

### Tower Matrices

Represent the effect of a linear optical element (polarizer,  $\lambda/4$ -wave, phase retarder, etc.) with  $2 \times 2$  matrices.

$\lambda/4$  wave plate



choose the length so that we get a phase change of  $\pi/2$

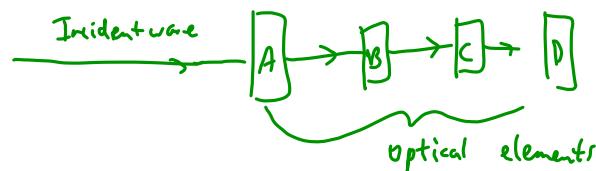
Incident light

$$\begin{pmatrix} A \\ B \end{pmatrix}$$

$$\text{After polarizer: } \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \frac{1}{2} \begin{pmatrix} A+B \\ A+B \end{pmatrix} = \frac{1}{2} (A+B) \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\text{After the } \lambda/4\text{-wave plate: } \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \frac{1}{2} (A+B) \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\frac{1}{2} (A+B) \begin{pmatrix} 1 \\ i \end{pmatrix} \rightarrow \text{Left circular polarization}$$



$$(\text{result}) = D C B A (\text{incident})$$

## Orthogonal Polarization

$$\vec{E}_1 \cdot \vec{E}_2^* = 0$$

You can always resolve arbitrary polarization  
into orthogonal components

$$\begin{pmatrix} A \\ B \end{pmatrix} = A \begin{pmatrix} 1 \\ 0 \end{pmatrix} + B \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{2} (A+iB) \begin{pmatrix} 1 \\ i \end{pmatrix} + \frac{1}{2} (A-iB) \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$\downarrow$                      $\downarrow$                      $\downarrow$                      $\downarrow$   
horizontal      vertical      left circular      right circular

### Eigenvectors of Jones Matrices

A particular polarization that emerges w/ same polarization it had at the beginning.

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \lambda \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

optical element

↓ can be complex

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \lambda \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$\begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = 0$$

Non trivial solution

$$\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$$

$$(a-\lambda)(d-\lambda) - bc = 0 \quad \text{get 2 roots } \lambda_1, \lambda_2$$

### Example

Right circular polarizer

$$\frac{1}{2} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$$

$$\underline{\lambda_1 = 1}$$

$$\left(\frac{1}{2} - 1\right) A + \frac{i}{2} B = 0$$

$$-\frac{1}{2} A = \frac{-i}{2} B$$

$$A = iB$$

$$B = -iA$$

$$\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

Right circular polarization  
is unaffected ( $\lambda=1$ )

$$\underline{\lambda_2 = 0}$$

$$\left(\frac{1}{2} - 0\right) A + \frac{i}{2} B = 0$$

$$\frac{1}{2} A = -\frac{i}{2} B$$

$$A = -iB$$

$$B = iA$$

$$\begin{pmatrix} 1 \\ i \end{pmatrix}$$

Left circular  
polarization gets  
killed  $\lambda=0$ .