Homework 5, due Tuesday May 2

1. Let A, B and C be operators. Show that
$$[A, BC] = B[A, C] + [A, B]C$$

2. The trace of an operator is the sum of the diagonal elements of its representation matrix:
$$\mathrm{Tr}A = \sum_n A_{nn}$$
Show that $\mathrm{Tr}AB = \mathrm{Tr}BA$

3. Let $A(t)$ be a matrix that depends on time and B a constant matrix. Suppose that
$$\frac{dA(t)}{dt} = A(t)B.$$
Show that $A(t) = A(0)\exp(Bt)$. What is the solution of
$$\frac{dA(t)}{dt} = BA(t)?$$

4. We showed in class that $\hat{N}(\hat{a}|n+1\rangle) = n(\hat{a}|n+1\rangle)$. Show that this implies that $\hat{a}|n+1\rangle = c|n\rangle$ where c is a constant. Similarly $\hat{a}^\dagger|n\rangle \propto |n+1\rangle$.

5. Prove that $\hat{a}^\dagger|n\rangle = e^{i\alpha}\sqrt{n+1}|n+1\rangle$ where α is an arbitrary phase factor.

6. We might as well take the phase factor α to be zero. Then, assuming that the vectors $|n\rangle$ are normalized, show that $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$ and
$$|n\rangle = \frac{1}{\sqrt{n!}}(\hat{a}^\dagger)^n|0\rangle.$$