MACS 332 - July 12, 2006
Quiz I - 30 minutes

NAME: Key.

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. Please enclose your final answers in boxes.

1. (10 Points)

a. Suppose $\mathbf{A}$ is invertible. Explain why $\mathbf{A}^T\mathbf{A}$ is invertible and show that $\mathbf{A}^{-1} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$.

A invertible ⇒ A^T invertible. The product of invertible matrices are invertible so A^TA is invertible

$A^{-1} = (A^TA)^{-1}A^T = A^{-1}(A^T)^{-1}A^T = A^{-1}I = A^{-1}$ ✓

b. Assume that for $\mathbf{A}_{n\times n}$ and some $\mathbf{b} \in \mathbb{R}^n$ the equation $\mathbf{Ax} = \mathbf{b}$ has non-unique solutions. List three properties of the matrix $\mathbf{A}$.

1. The columns of A ~~has~~ are linearly dependent.
2. $\det(A) = 0$
3. There are less than n-pivots in A.

2. (10 Points) Calculate the determinant of $\mathbf{A}$.

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{bmatrix}$$

$\det(A) = 1 \cdot \det\begin{pmatrix} 1 & 1 \\ 4 & 2 \end{pmatrix} - 3\det\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} + 5\det\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} =$

$= 1(2-4) - 3(4-3) + 5(8-3) = -2 - 3 + 25 = \boxed{20 = \det(A)}$

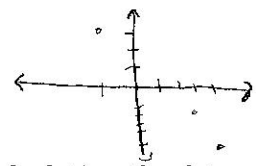
3. (10 Points) Determine the LU-decomposition of **A**.

$$\mathbf{A} = \begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}$$

$$A \sim \begin{bmatrix} 3 & -6 & 3 \\ 0 & -7+12 & 2-6 \\ 0 & 7-2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 5 & 1 \end{bmatrix} \sim \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 0 & +5 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1/3 & 1 & 1 \end{bmatrix} \qquad LU = \begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix} \checkmark$$

4. (10 Points) Given that the points,



$$p_1 = (0,0), \quad p_2 = (-1,3), \quad p_3 = (4,-5), \quad p_4 = (3,-2),$$

are the vertices of a parallelogram. Determine the area of this parallelogram by calculating the determinant of the appropriate matrix **A**.

$$\overrightarrow{P_1P_2} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \quad \overrightarrow{P_1P_3} = \begin{bmatrix} 4 \\ -5 \end{bmatrix} \Rightarrow A = \begin{bmatrix} -1 & 4 \\ 3 & 5 \end{bmatrix} \Rightarrow |\det(A)| = |(-5-12)| = 7$$

The parallelogram has 7 units of Area.

— x —

$$\overrightarrow{P_1P_2} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \quad \overrightarrow{P_1P_4} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} \Rightarrow A = \begin{bmatrix} -1 & 3 \\ 3 & -2 \end{bmatrix} \Rightarrow |\det(A)| = |2-9| = 7$$

Taking any 2 vectors from these points, which are not parallel, will lead to the soln.