Approaches to practical calculations

- Theory/computation:
  - Solve Schroedinger equation, calculate dipole moment
  - Use Fermi's Golden rule to calculate transition rate
- Beam propagation:
  - Use cross-sections for absorption and stimulated emission to calculate exponential damping or growth of beam
- Rate equations
  - Einstein A and B coefficients
  - Set up equations for the populations of all participating energy levels
  - Couple these to equations for photons in beam

Wave propagation with absorption

- Consider light absorption from a thin slab

$$I_1 = I_0 - I_0 \alpha \Delta z$$

- Generalize to an equation for arbitrary length:

$$I_1 - I_0 = \Delta I = -I_0 \alpha \Delta z \rightarrow \frac{dI}{dz} = -\alpha I$$

$$I(z) = I_0 e^{-\alpha z}$$ Beer's Law

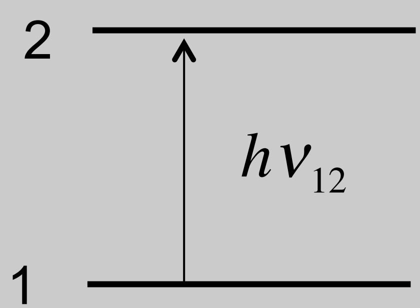
- Absorption coefficient (units m^{-1}) is proportional to the number density of absorbers:

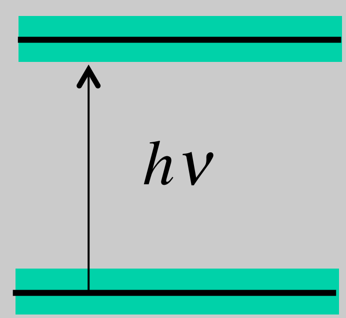
$$\alpha = N_1 \sigma$$

  – N_1 = number density (m^{-3}) of species in level 1
  – σ? Has units of m^2, = "cross-section"

Models for σ: hard and soft spheres

- Consider an collection of “black” spheres that absorb if struck by a photon.
- Cross-section for absorption is just the projected area of the sphere. $\sigma = \pi a^2$
- For an atom, the probability of absorption depends on how close the incident frequency is to resonance:





Absorption lines are *broadened*, so *exact* energy is not required.

$$\sigma \rightarrow \sigma(\nu)$$

Example: absorption of pump light in Nd:YAG

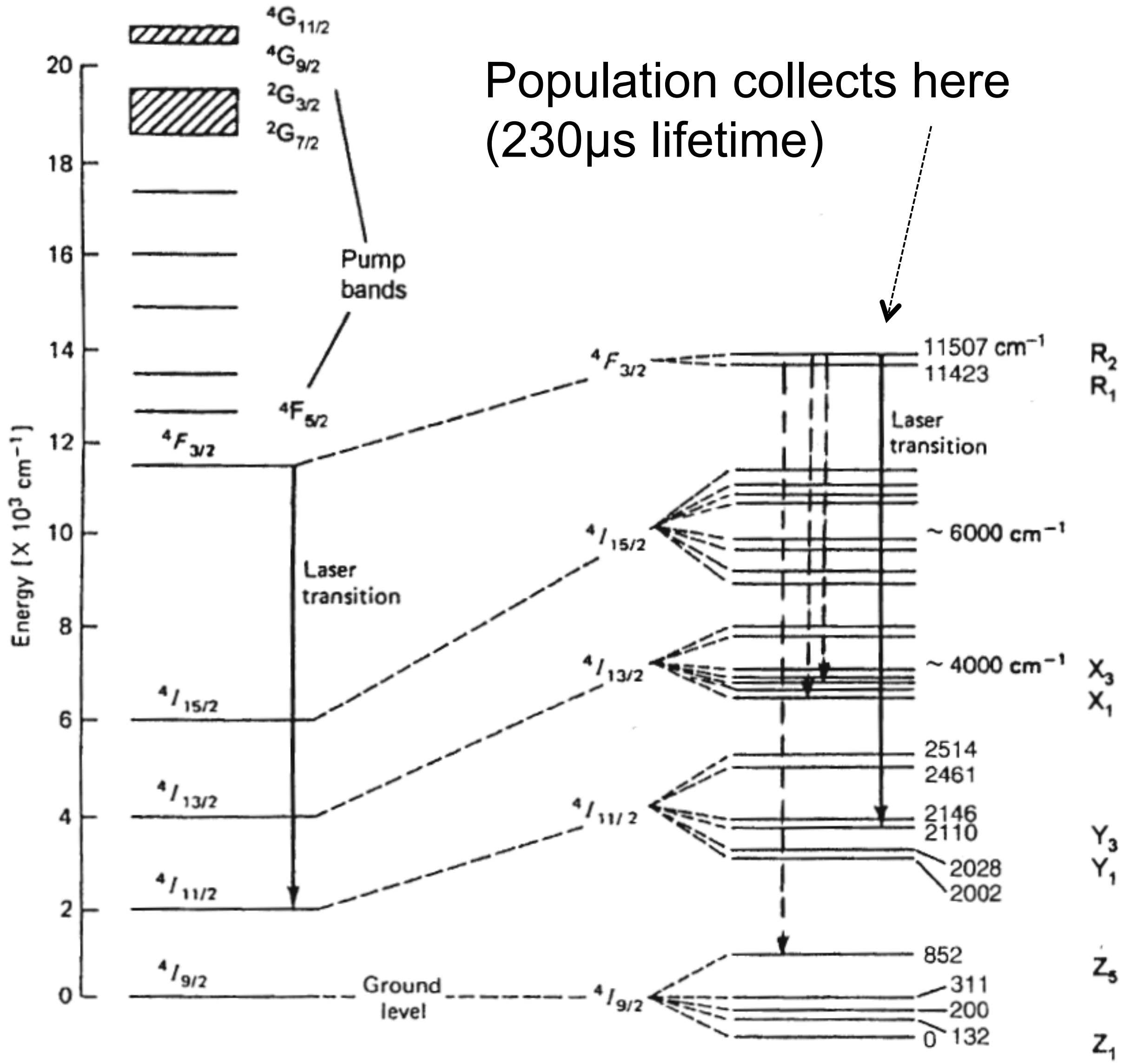
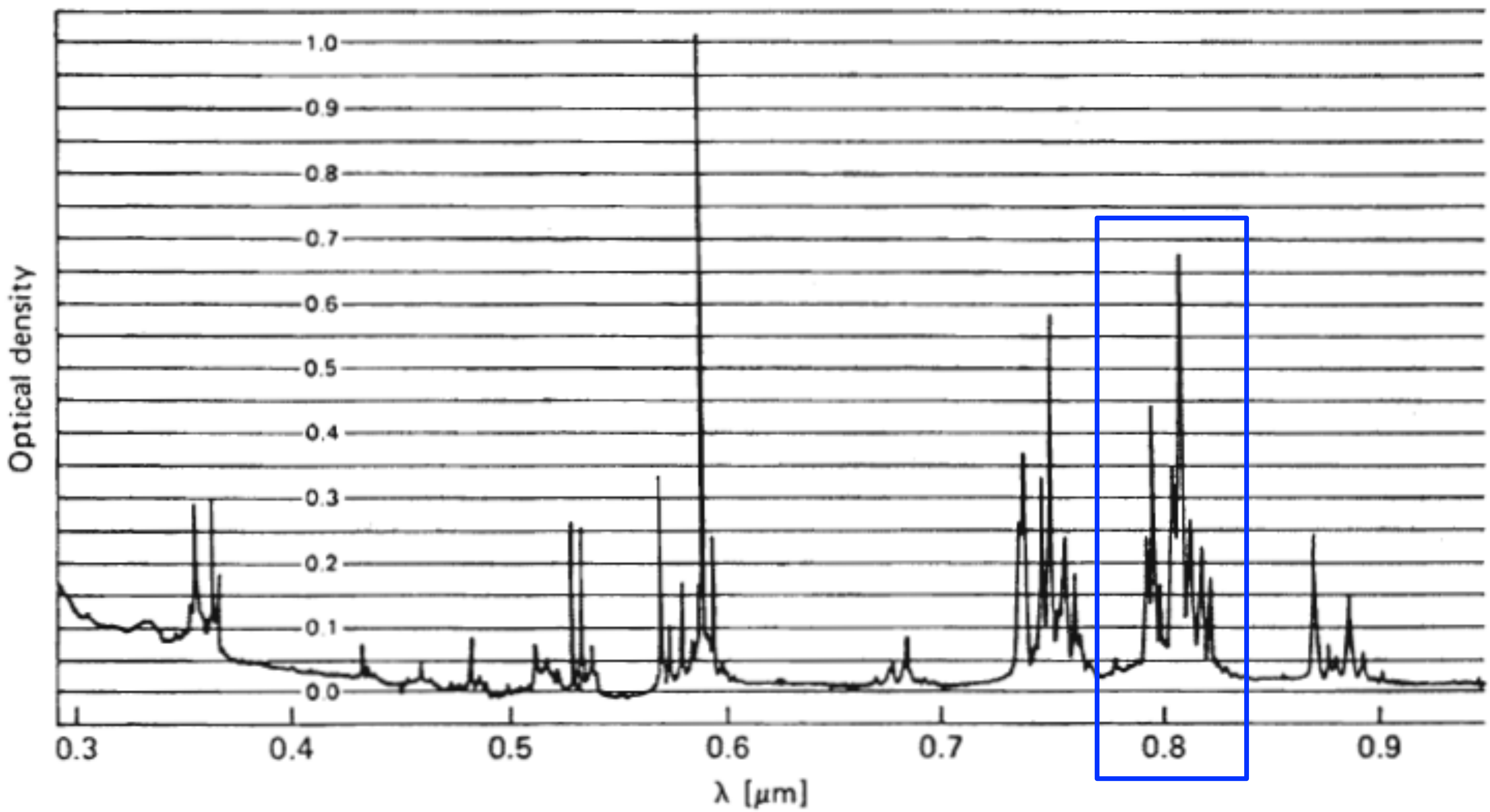


Fig. 2.2. Energy level diagram of Nd:YAG. The solid line represents the major transition at 1064 nm, and the dashed lines are the transitions at 1319, 1338, and 946 nm.

- Nd^{3+} is a heavy ion with many possible transitions
- Pump to anywhere above the $^4F_{3/2}$ level

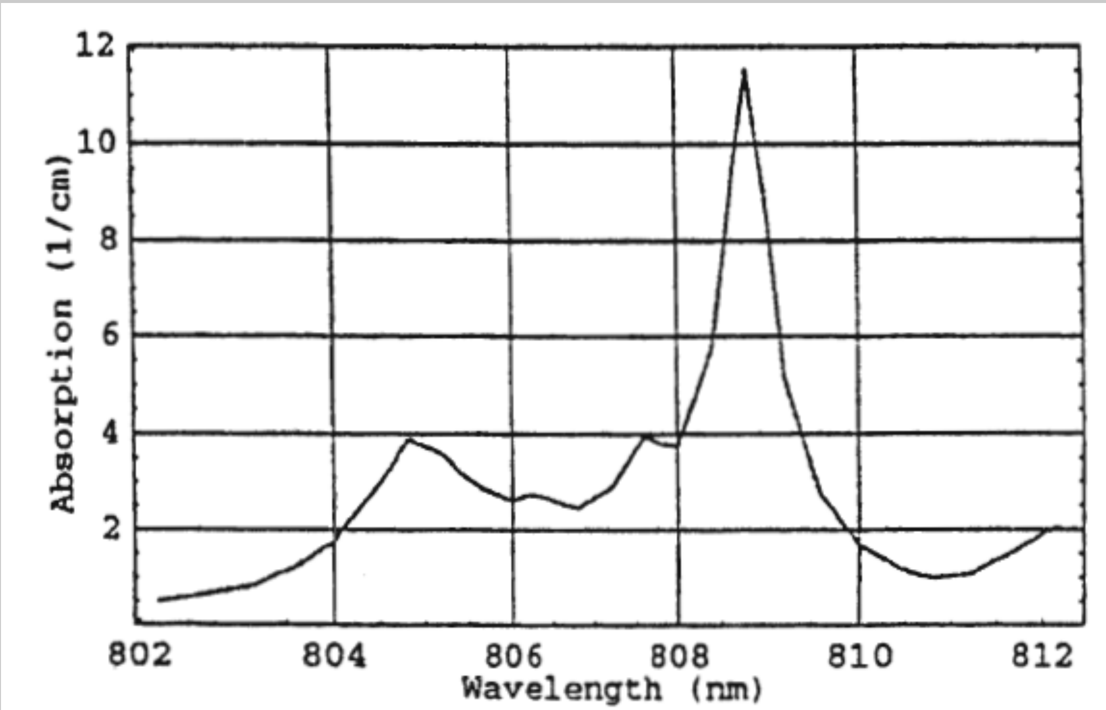
Absorption spectrum of Nd^{3+}:YAG



- Optical density (OD) = $-\log_{10}[T]$

Pump bands near 808nm

- Powerful laser diodes (LD) are available near 808nm



3mm thick Nd:YAG crystal

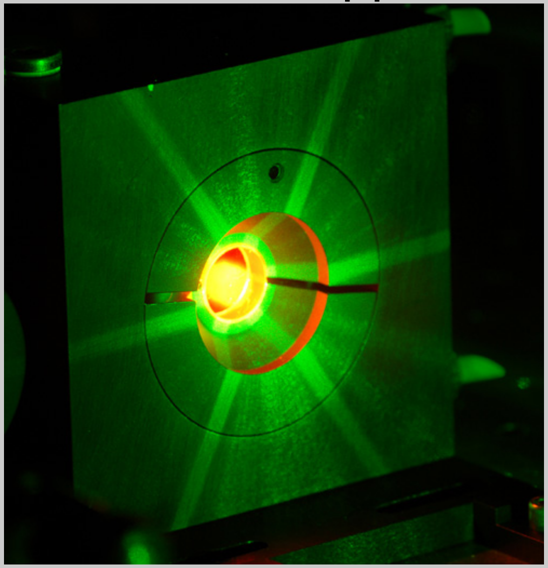
- What % is absorbed at the peak (α=11/cm)?
- What is the OD?
  $= -\log_{10}[T]$
- If $N_{Nd}=1.38 \times 10^{20}/cm^3$ (1% atomic), what is the absorption cross-section?

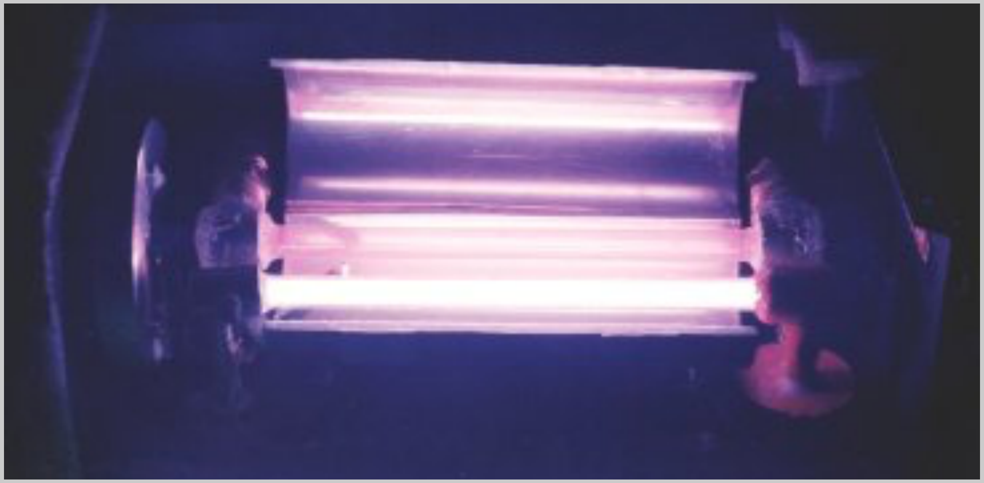
- Note: LD output wavelength depends on temperature, so temperature must be set and stabilized in real systems.

Amplifiers: pumping and small-signal gain

- Absorption $I[z] = I_0 \exp[-N_0 \sigma_{12} z] = I_0 \exp[-\alpha z]$

- Gain $I[z] = I_0 \exp[N_{inv} \sigma_{21} z] = I_0 \exp[g z]$
  - What is the inversion density?
  - How to express it in terms of the pump distribution
  - How does gain depend on λ or ω ?
  - What happens when the inversion density is depleted?





Simple gain calculation

- Assume spatially uniform pump distribution

$$G_0 = \exp\left[N_{inv}\sigma_{21} L\right]$$ Small-signal gain

- Available energy for extraction:

$$E_{stor} = N_{inv} A L h\nu_{21} \rightarrow N_{inv} = \frac{E_{stor}}{A L h\nu_{21}}$$

A = area of beam

$$G_0 = \exp\left[\frac{E_{stor}}{A}\frac{\sigma_{21}}{h\nu_{21}}\right]$$

- Energy fluence = energy per unit area
- Define:
  - "stored fluence" $\Gamma_{stor} = \frac{E_{stor}}{A}$
  - "saturation fluence" $\Gamma_{sat} = \frac{h\nu_{21}}{\sigma_{21}}$

$$G_0 = \exp\left[\frac{\Gamma_{stor}}{\Gamma_{sat}}\right]$$

Example: Ti:sapphire saturation fluence

Saturation fluence

$$\Gamma_{sat} = \frac{h\nu_{21}}{\sigma_{21}} \ \frac{\mathrm{J}}{\mathrm{cm}^2}$$

For Ti:sapphire:

– λ_{21}=800nm, $h\nu_{21}$ = 1.55eV = 2.48x10^{-19} J

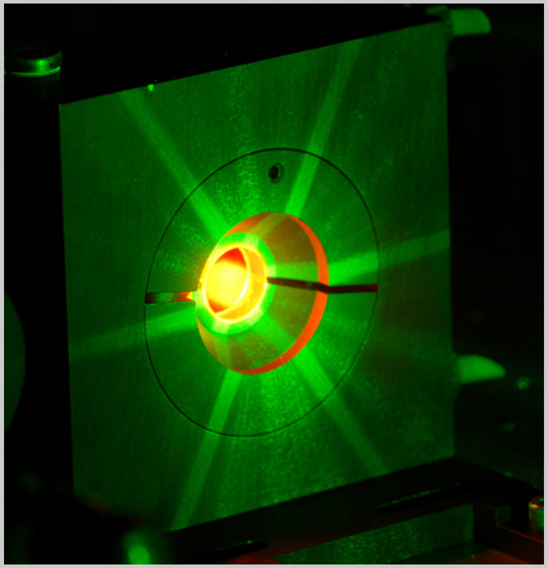
– σ_{21} = 2.8 x 10^{-19} cm^2

Γ_{sat} = 0.85 J/cm^2

Use this in gain calculation:

$$G_0 = \exp\left[\frac{\Gamma_{stor}}{\Gamma_{sat}}\right]$$

Saturation fluence varies with gain medium



Example: Ti:sapphire amplifier

- Pump laser has 10mJ per pulse, calculate spot size in crystal for $G_0 = 5$

$$G_0 = \exp\left[\frac{\Gamma_{stor}}{\Gamma_{sat}}\right] \qquad \Gamma_{sat} = 0.85\ \text{J/cm}^2$$

- For $G_0 = 5$ we can calculate the required stored fluence:

$$\Gamma_{stor} = \Gamma_{sat} \ln[G_0] = 1.37\ \text{J/cm}^2$$

- Incident fluence must be larger b/c of wavelength ratio

$$\frac{h\nu_p}{h\nu_L} = \frac{\lambda_L}{\lambda_p} = \frac{800nm}{532nm} \approx 1.5 \qquad \Gamma_{inc} = (\lambda_L/\lambda_p)\,\Gamma_{stor} = 2.06\ \text{J/cm}^2$$

- We have 10 mJ incident (assuming all is absorbed)

Total stored energy = 6.7 mJ
$A = 4.85 \times 10^{-3}$ cm^2
$w_0 = 390$ μm

Tells us what size to focus the pump beam

"Small-signal" gain

- We calculated G_0 = 5: this is the "small-signal gain"
  - Energy of input pulse: 1 μJ initial stored: 6.7mJ
  - Energy of output pulse: 5 μJ final stored: 6.696mJ

- What if we have more energy input?
  - Energy of input pulse: 1 mJ initial stored: 6.7mJ
  - Energy of output pulse: 5 mJ final stored: 2.7mJ

  - Energy of input pulse: 5 mJ initial stored: 6.7mJ
  - Energy of output pulse: 25 mJ final stored: -13.3mJ
- We will need to account for **saturation** of gain.