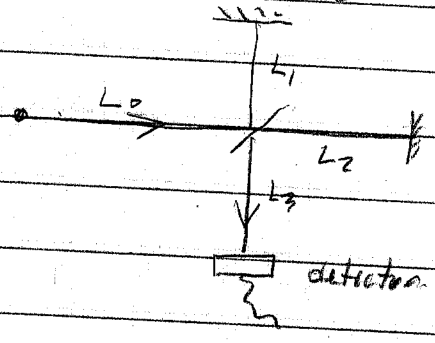
Michelson interferometer



Assume 50-50 beamsplit

input $I_0 = \frac{c}{8\pi} E_0^2$

intensity is split 50%

$\rightarrow E_1 = \frac{1}{\sqrt{2}} E_0 = E_2$

2 methods

1) optical path difference -

$$E_1 = E_0 e^{ik_0L_0} \cdot \frac{1}{\sqrt{2}} e^{2ik_0L_1} \frac{1}{\sqrt{2}} e^{ik_0L_3} e^{-i\omega t}$$

$$= \frac{1}{2} E_0 \exp(ik_0(L_0 + 2L_1 + L_3))\, e^{-i\omega t}$$

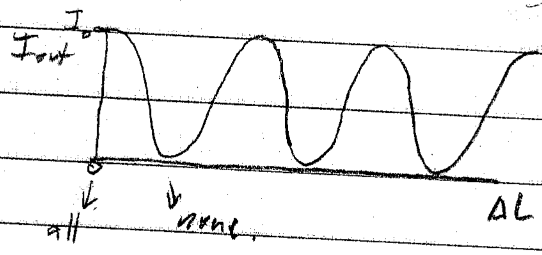
$$E_2 = \frac{1}{2} E_0 \exp(ik_0(L_0 + 2L_2 + L_3))\, e^{-i\omega t}$$

$$E_{out} = E_1 + E_2 = \frac{1}{2} E_0 e^{ik_0(L_0+L_3)} \left(e^{2ik_0L_1} + e^{2ik_0L_2}\right)$$

Note similarity to method when working with polarization.

$\rightarrow \Delta\phi = \delta = 2k_0(L_1 - L_2)$

$$I_{out} = \frac{1}{4} \frac{c}{8\pi} E_0^2 \left|1 + e^{i\delta}\right|^2 = \frac{1}{2} I_0 \left(1 + \cos 2k_0(L_1 - L_2)\right)$$



I_0 is indep of common phase

2) time delay

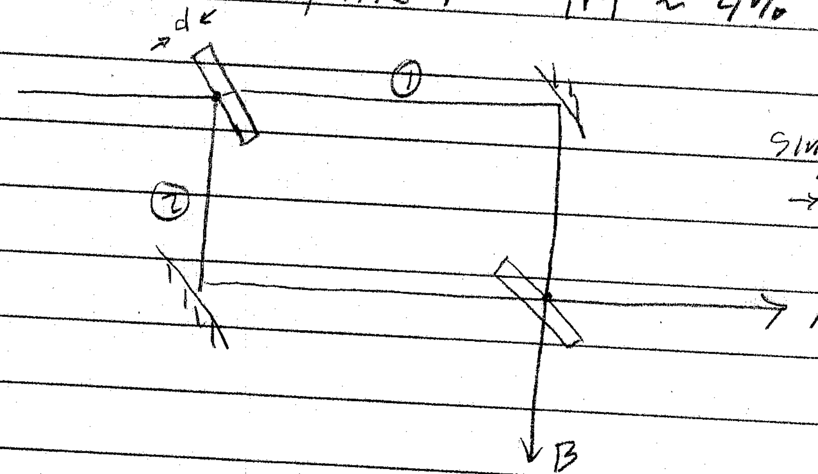
path 1 $\quad \tau_1 = (L_0 + 2L_1 + L_3)/c$

path 2 $\quad \tau_2 = (L_0 + 2L_2 + L_3)/c$

then $\Delta\phi = \frac{\omega}{c}(2L_1 - 2L_2) = k_0(2L_1 - 2L_2)$ same.

Where does energy go? Out other port.
Mach-Zender interferometer: can measure both ports.

windows as beamsplitters $|r|^2 \sim 4\%$



single pass thru
$\rightarrow \phi_w = k_o n d \cos 45°$

$$E_{1A} = E_o \sqrt{1-R}\,(+1)\, e^{i\phi_w} e^{ik_oL_1} \sqrt{R}\,(-1)$$
$$E_{2A} = E_o \sqrt{R}\,(-1)\, e^{ik_oL_2} e^{i\phi_w} \sqrt{1-R}\,(+1)$$
$$\rightarrow \delta = k_o(L_1 - L_2) \quad \text{same amplitude.}$$
$$E_{1B} = E_o \sqrt{1-R}\,(+1)\, e^{i\phi_w} e^{ik_oL_1} \sqrt{1-R}\,(+1)\, e^{i\phi_w}$$
$$= E_o (1-R)\, e^{i2\phi_w} e^{ik_oL_1}$$
$$E_{2B} = E_o \sqrt{R}\,(-1)\, e^{ik_oL_2} e^{i2\phi_w} (+1) \sqrt{R}$$
$$= E_o R\, e^{i2\phi_w} \left(-e^{ik_oL_2}\right)$$

$$I_A = I_o \cdot R(1-R)\left(1 + \cos k_o(L_1 - L_2)\right) \qquad \delta = \pi,\ I_A = 0$$

$$I_B = \frac{c}{8\pi} E_o^2 \left|(1-R) - R e^{ik(L_2 - L_1)}\right|^2$$

$$= I_o \left((1-R)^2 + R^2 - (1-R)R\left(e^{ik_o(L_2-L_1)} + e^{-ik_o(L_2-L_1)}\right)\right)$$
$$= I_o \left(1 - 2R + 2R^2 - 2(1-R)R\cos\delta\right)$$
$$I_B = I_o\left(1 - 2(1-R)R(1 + \cos\delta)\right) \qquad \delta = \pi,\ I_B = I_o$$

Angular tilt

set $\Delta L = 0$

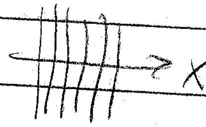
$$E_1 = \frac{1}{2} E_0 e^{ik_0 x \sin\theta_0}$$

$$E_2 = \frac{1}{2} E_0$$

$$I_{out}(x) = \frac{1}{4} I_0 \left| 1 + e^{ik_0 x \sin\theta_0} \right|^2$$

$$= \frac{1}{2} I_0 \left(1 + \cos(k_0 x \sin\theta_0)\right)$$

→ fringes in output



small θ_0 $\sin\theta_0 \to \theta_0$

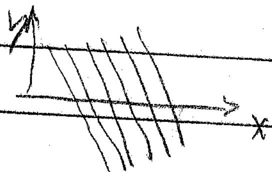
$$I_{out}(x) = \frac{1}{2} I_0 \left(1 + \cos(k_0 x \theta_0)\right)$$

tilt in x, y

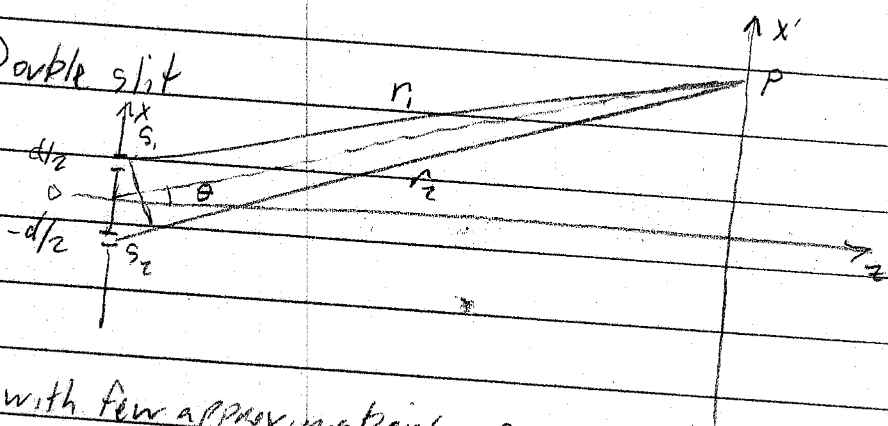
$$E_1 = \frac{1}{2} E_0 e^{ik_0 x \theta_x} e^{ik_0 y \theta_y}$$

$$\to I_{out}(x,y) = \frac{1}{2} I_0 \left(1 + \cos(k_0 (x\theta_x + y\theta_y))\right)$$

rotated fringes.



Double slit



with few approximations, sources at S_1, S_2 are either

points $E \propto \frac{1}{r} e^{ikr}$

or lines $E \propto \frac{1}{\sqrt{r}} e^{ikr}$

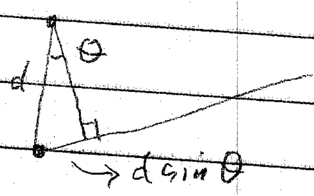
the leading factor controls the brightness of the pattern.
the exponential terms will give the structure of the pattern.

$E_1 \propto e^{ik_0 r_1}$ $\qquad r_1 = \overline{S_1P}$

$E_2 \propto e^{ik_0 r_2}$ $\qquad r_2 = \overline{S_2P}$

$\delta = k_0 (r_2 - r_1)$

for small angles, treat each ray as emerging at the same angle: $\theta_1 \sim \theta_2 \sim \theta$



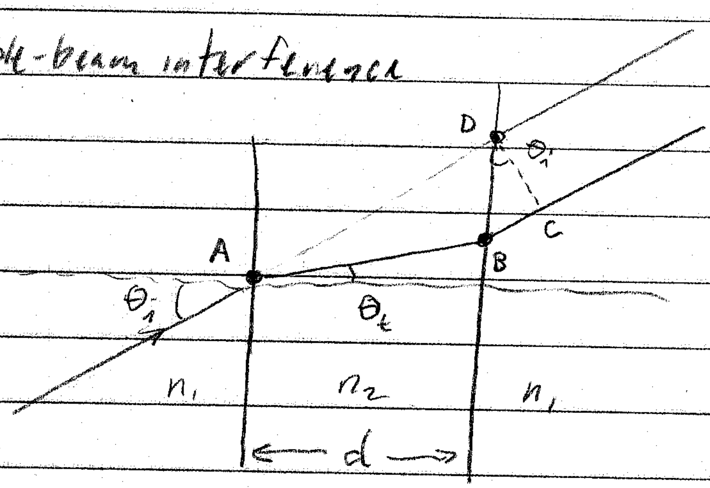
$\rightarrow \delta \approx k_0 d \sin\theta$

valid for small angles.

A more rigorous calculation: calc. exact geometry, then make approximations. $\rightarrow$ next orders of approx

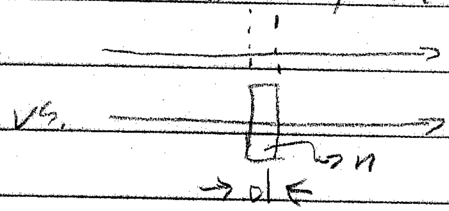
$\rightarrow$ condition on range of θ approximation is valid

Double-beam interference



Suppose we introduce a tilted window into one arm of an interferometer. What is the phase shift?

simple case:



$S_1 = d$

$S_2 = nd$

$\Delta S = (n-1)d$

$\Delta\phi = k_0(n-1)d$

now tilt:

from diagram above:

try this: path in glass is $d/\cos\theta_t = \overline{AB}$

$\rightarrow \Delta S = \dfrac{n_2 d}{\cos\theta_t} - n_1 d$ X!

this is wrong! We must be more careful.

geometric method: measure all the way out to line ⊥ to external ray

$S_1 = n_1\overline{AD} \qquad = n_1 d/\cos\theta_i$

$S_2 = n_2\overline{AB} + \overline{BC} \quad = n_2 d/\cos\theta_t + n_1\overline{BD}\sin\theta_i$

$\overline{BD} = d\tan\theta_i - d\tan\theta_t$

$$S_2 = \frac{n_2 d}{\cos\theta_t} + n_1 d(\tan\theta_i - \tan\theta_t)\sin\theta_i$$

$n_1\sin\theta_i = n_2\sin\theta_t$

$$s_2 = \frac{n_2 d}{\cos\theta_t} - \frac{n_2 d \sin^2\theta_t}{\cos\theta_t} + n_1 d \sin\theta_i \tan\theta_i$$

$$= n_2 d \cos\theta_t + n_1 d \frac{\sin^2\theta_i}{\cos\theta_i}$$

$$s_2 - s_1 = n_2 d \cos\theta_t + n_1 d \frac{\sin^2\theta_i}{\cos\theta_i} - n_1 d \frac{1}{\cos\theta_i}$$

$$= n_2 d \cos\theta_t - n_1 d \cos\theta_i$$

wave method:

$$E_1(y,z) = E_0 e^{i(k_{1y} y + k_{1z} z)}$$

where $k_{1y} = k_0 n_1 \sin\theta_i$

$k_{1z} = k_0 n_1 \cos\theta_i$

$$E_2(y,z) = E_0 e^{i(k_{2y} y + k_{2z} z)}$$

$k_{2y} = k_0 n_2 \sin\theta_t$

$k_{2z} = k_0 n_2 \cos\theta_t$

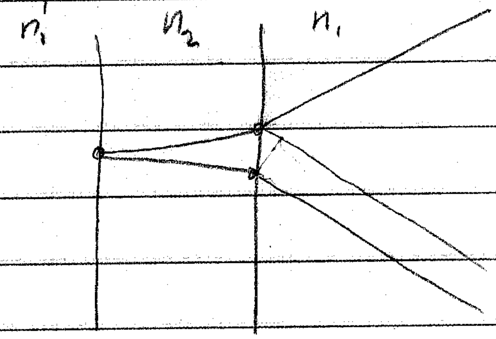
but $k_{1y} = k_{2y}$

evaluate waves at $z = d$

$$E_1 + E_2 = E_0 e^{i k_{1y} y} \left[e^{i k_0 n_1 d \cos\theta_i} + e^{i k_0 n_2 d \cos\theta_t} \right.$$

can see $\Delta\phi = k_0 d (n_2 \cos\theta_t - n_1 \cos\theta_i)$

Double pass



$\Delta\phi = 2k_0 n_2 d\cos\theta_t$

$+\pi$

bk of reflection phase change.

let $\Lambda = 2n_2 d\cos\theta_t$ (optical path)

Fizeau fringes / Newton's rings

r, t : from n_1 to n_2 $\quad$ r', t' from n_2 to n_1

1st reflection $\quad E_1 = E_0 r$

2nd reflection $\quad E_2 = E_0 t r' t' e^{ik_0\Lambda}$

$r' = -r$

$tt' = 1 - r^2$

from Fresnel eqns : see F-P notes.

$$I_{out} \propto |E_1 + E_2|^2$$
$$= I_0 |r|^2 \left| 1 - (1-r^2) e^{ik_0\Lambda} \right|^2$$
$$= I_0 |r|^2 \left(1 + |1-r^2|^2 - 2(1-r^2)\cos k_0\Lambda \right)$$

At $k_0\Lambda = m \cdot 2\pi$ $\quad \cos(k_0\Lambda) = +1 \rightarrow$ dark fringe.

$k_0\Lambda = m \cdot 2\pi + \pi \quad \rightarrow$ bright fringe.

If d is slowly varying $d(x) \rightarrow$ series of fringes.

spacing: $\Lambda = \lambda_0$ total opt. path change $= 1\lambda_0$