Radiation w/ velocity ∥ acceleration

$$\vec{E}_a = \frac{e}{c^2 K^3 R^3} \vec{R}\left(\left(\vec{R} - \cancel{\vec{\beta} R}\right) \times \vec{a}\right)$$

0 since here $\vec{\beta} \times \vec{a} = 0$

now $\vec{S}_a = \vec{S}_a(\beta = 0) \cdot K^{-6}$

but this is power radiated from charge:

$$\frac{d\mathcal{E}_{tot}}{dt} = -\int \vec{S}_a \cdot d\vec{A}$$

we want radiated power measured at distant point.

$$P = -\frac{d\mathcal{E}_{tot}}{dt'} = -\frac{d\mathcal{E}}{dt}\frac{dt}{dt'} \qquad t' = t - R(t')/c$$

↳ retarded time

$$\frac{dt}{dt'} = 1 + \frac{1}{c}\frac{d}{dt'}\left|\vec{r} - \vec{r}_e(t')\right| = 1 + \sum \frac{u_i}{c}\frac{d}{dx_{ei}}\left|\vec{r} - \vec{r}_e\right|$$

$$\left|\vec{r} - \vec{r}_e\right| = \sqrt{r^2 + r_e^2 - 2\vec{r}\cdot\vec{r}_e}$$

$$\frac{d}{dx_{ei}}\left|\vec{r} - \vec{r}_e\right| = \frac{1}{2}\frac{1}{\sqrt{}}\left(2x_{ei} - 2x_i\right)$$

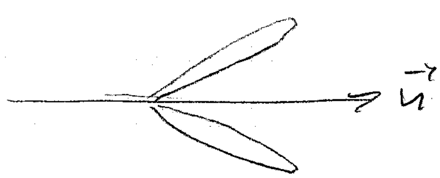
$$\sum \beta_i \frac{d}{dx_{ei}}\left|\vec{r} - \vec{r}_e\right| = \sum \frac{\beta_i (x_{ei} - x_i)}{R} = -\frac{\vec{\beta}\cdot\vec{R}}{R}$$

$$\frac{dt}{dt'} = 1 - \frac{\vec{\beta}\cdot\vec{R}}{R} = K = 1 - \beta\cos\theta$$

↳ measured rel to $\vec{u}$

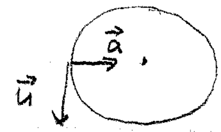
now we have K^{-5} in $dP/d\Omega$

this directs lobes toward $\vec{u}$

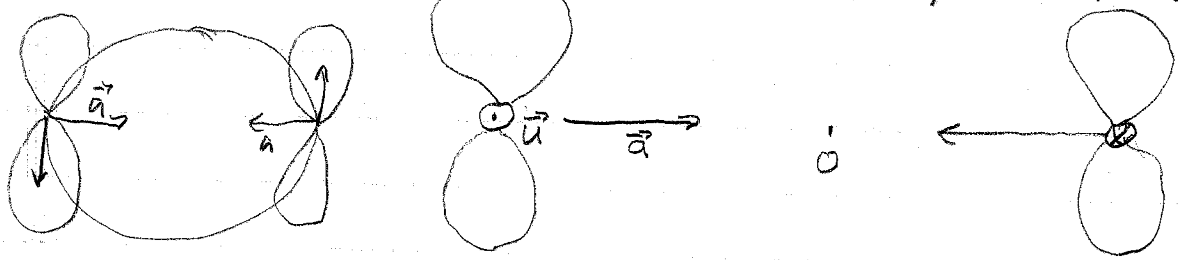


Synchrotron radiation - circular orbit.

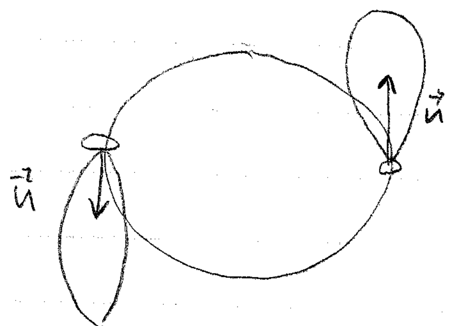
$\vec{u} \perp \vec{a}$



At low speeds, Larmor formula w/ θ relative to $\vec{a}$



higher speeds: push lobes forward, toward $\vec{u}$



"searchlight"

output: pulses w/ rep rate 1/orbit period

Antennas - radiation from collections of charges

2 assumptions:

1) localized charge/current distribution (size d)

2) source oscillates w/ frequency ω $(\lambda = 2\pi c/\omega)$

Observation distance ~ r

Must make approximations to do calculations: order these dimensions

Scaling analysis:

$$\vec{E} = e\left[\frac{(\hat{R}-\beta)(1-\beta^2)}{K^3R^2} + \frac{\hat{R}\times((\hat{R}-\vec{\beta})\times\vec{a})}{c^2K^3R}\right]$$

$$\vec{B} = \hat{R}\times\vec{E}, \quad \vec{S} = \frac{c}{4\pi}(\vec{E}\times\vec{H})$$

compare relative magnitude of terms.

K^3 common to all

single charge: oscillates over dist d at freq. $\nu =$

$u \sim d\cdot\nu$ $\quad \beta \sim d\nu/c \sim d/\lambda$ $\quad a/c^2 \sim d/\lambda^2$

since $\beta < 1$, single charge must have $d < \lambda$

example: $\nu \sim 1$ MHz (AM radio) | 100 MHz FM

$\rightarrow \lambda \sim 300$ m | 3 m

$d \sim 1$ cm

$u \sim d\nu \sim 10^{-2}\text{ m}\cdot 10^6\text{ s}^{-1} \sim 10^4$ m/s | 10^6 m/s

$\rightarrow \beta \sim d/\lambda \sim 3\times10^{-5}$ | 3×10^{-3}

$$E \sim \frac{1}{R^2} + \frac{a}{c^2R} \sim \frac{1}{R^2} + \frac{d}{\lambda^2 R}$$

$$S \sim \frac{1}{R^4} + \frac{2d}{\lambda^2R^3} + \frac{d^2}{\lambda^4R^2} \sim 1, \ \frac{2dR}{\lambda^2}, \ \frac{d^2R^2}{\lambda^4}$$

Antennas - radiation from collections of charges.

consider charge distribution that is:
- localized
- oscillating with frequency ω

$\rightarrow$ 3 length scales

size: d observation distance: r

wavelength of radiation: $\lambda = 2\pi c/\omega$

Ordering of these scales $\rightarrow$ method for solution.

<u>Quasistatic</u>

low frequency $\rightarrow$ long wavelength

e.g. $\nu = 1\,MHz$ (AM radio)

$\lambda = 300\,m$

observe w/in lab: $r \lesssim 3\,m$

$r \ll \lambda$

suppose source is $d \sim 1\,cm$

Can we ignore retardation effects? yes

$d \ll \lambda$

if charges are moving across d, $u \sim d\cdot\nu \sim 10^{-2}\cdot 10^{6}$

$= 10^{4}\,m/s$

$\frac{u}{c} \sim \frac{d\cdot\nu}{c} \sim \frac{d}{\lambda}$

$u \ll c$

$\beta \ll 1$

$a \sim d\cdot\nu^2$

$K \equiv 1 - \hat{R}\cdot\vec{\beta} \approx 1$

$$E \sim e\left[\frac{(\hat{R}-\vec{\beta})(1-\beta^2)}{K^3R^2} + \frac{\hat{R}\times((\hat{R}-\vec{\beta})\times\vec{a})}{c^2K^3R}\right]$$

$$\sim e\left[\frac{1}{R^2} + \frac{d\cdot\nu^2}{c^2 R}\right] \sim e\left[\frac{1}{R^2} + \frac{d}{\lambda^2 R}\right]$$

Now we are left with static fields (e.g. coulomb) that are slowly varying.

$\rightarrow \Phi(\vec{r},t) \approx \int \frac{\rho(\vec{r}\,',t)}{|\vec{r}-\vec{r}\,'|} d^3r'$

if $r \gg d$ we can do a multipole expansion
e.g. calc. dipole, quadrupole moment.
fields $\sim 1/R^2$ or faster.

<u>Radiation fields</u>:

if $r \gg \lambda$, static fields drop off quickly
$\rightarrow$ radiation

if source is small, $d \ll \lambda$ ignore retardation w/in source.

larger sources $d \sim \lambda$: superposition