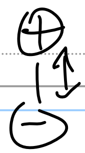
Note Title 7/10/2006



Lorentz force $\quad \underline{q\vec{E} + q\,\vec{v}\times\vec{B}} \Rightarrow$ potential formulation

$\qquad\qquad\qquad\qquad\quad \phi \qquad\qquad \vec{\nabla}\times\vec{A}$

identity 4

$$\vec{\nabla}(\vec{A}\cdot\vec{B}) = \vec{A}\times\cancel{(\vec{\nabla}\times\vec{B})} + \vec{B}\times(\vec{\nabla}\times\vec{A}) + \cancel{(\vec{A}\cdot\vec{\nabla})\vec{B}} + (\vec{B}\cdot\vec{\nabla})\vec{A}$$

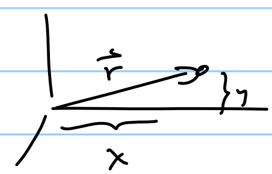
$\vec{v}\times\vec{\nabla}\times\vec{A}$

v   velocity   v   v   vector potential

$\frac{\partial}{\partial x} v(t)$

⇒ Canonical mom.

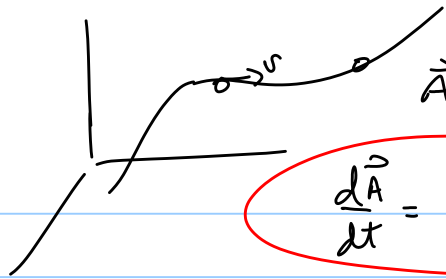
$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$



$$\vec{r} = x(t)\,\hat{x} + \_\_\,\hat{y} + \_\_\,\hat{z}$$

$$x^2\,\hat{x} + xy\,\hat{y}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{v}(t)$$



variation of $\vec{A}$ wrt time but due spatial variations

$$\frac{d\vec{A}}{dt} = \left.\frac{\partial \vec{A}}{\partial t}\right|_{\text{fixed position}} + (\vec{v}\cdot\vec{\nabla})A$$

$$\frac{d\vec{A}}{dt} = \frac{\partial A}{\partial x} v_x dt + \frac{\partial A}{\partial y} v_y dt + \cdots \quad \frac{\partial A}{\partial t} dt$$

$v_y = \frac{dy}{dt}$

$$\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$$

$x(t)$

$\vec{r}(t)$

$\frac{\partial}{\partial x}\vec{r} = 0?$

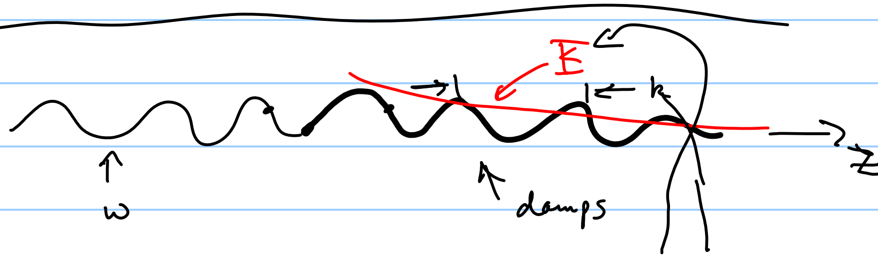
0 r doesn't change with time at fixed position

$$\underset{\text{total derivative}}{\frac{d\vec{r}}{dt}} = \left.\cancel{\frac{\partial \vec{r}}{\partial t}}\right|_{\text{fix position}} + \left(v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z}\right)(x\hat{x} + y\hat{y} + z\hat{z})$$

$$= v_x\hat{x} + v_y\hat{y} + v_z\hat{z} = \vec{v} = \frac{d\vec{r}}{dt}$$

$$v_x \frac{\partial}{\partial x}(x\hat{x} + y\hat{y} + z\hat{z}) = v_x\hat{x}$$

9.7



(a) PDE $T\frac{\partial^2 \tilde f}{\partial z^2} = \mu \frac{\partial^2 \tilde f}{\partial t^2} + \gamma \frac{\partial \tilde f}{\partial t}$

(b) $\tilde f(z,t) = \tilde F(z)\, e^{-i\omega t}$ put into PDE

assume

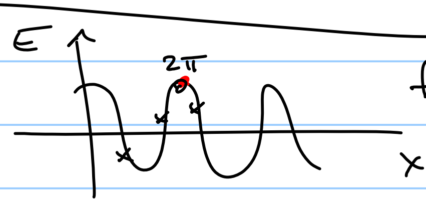
$\Rightarrow$ ODE in $\tilde F(z) \propto e^{ikx}$ no damping

$e^{-Bz}\, e^{ikz}$ with damping

Soln e^{iKz} where K is complex $K = i\bar{K} + k$

$= \text{Re}\,K + \text{Im}\,K$

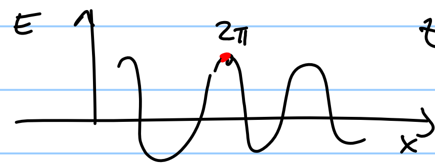
$E_0 \cos(kx - \omega t)$



fix $t=0$

phase $\varphi(x,t) = \#$

$2\pi = kx$ $t=0$ $x = \frac{2\pi}{k}$



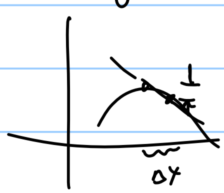
$t>0$

$2\pi = kx - \omega t$ $x = \frac{2\pi + \omega t}{k}$

$t>0$

speed of wave crest: to follow a wave crest

$\delta\varphi = 0 = \delta\varphi(x,t) = \frac{\partial\varphi}{\partial x}dx + \frac{\partial\varphi}{\partial t}dt$



$f(x)$ $\delta f = \frac{\partial f}{\partial x}dx$

$k\,dx = \omega\,dt$

Velocity of wave crest $= \frac{dx}{dt} = \frac{\omega}{k} = \frac{2\pi\nu}{2\pi/\lambda} = \lambda\nu = v_{phase}$

$\varphi(x,t)$ $\quad \left.\frac{\partial x}{\partial t}\right|_{\varphi} = \frac{-(\partial\varphi/\partial t)_x}{(\partial\varphi/\partial x)_t} = \frac{\omega}{k}$

$PV = nRT$ $\qquad$ $\varphi = kx - \omega t$

3 variables $\qquad$ 3 variables

Dispersion relation $\qquad \lambda\nu = c \qquad c = \frac{\omega}{k} \qquad \omega = ck$

light in vac

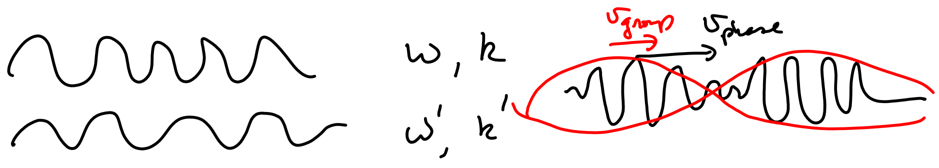
$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

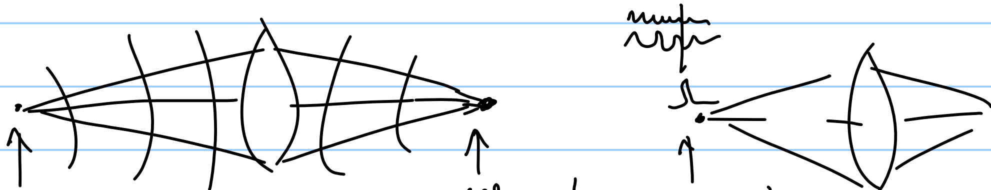
$$E = E_0 e^{i(kx-\omega t)}$$

Sch. eqn $\quad$ PDE $\quad \frac{\partial^2\psi}{\partial x^2} \propto \frac{\partial\psi}{\partial t} \Rightarrow$ plug $\psi = \psi_0 e^{i(kx-\omega t)} \Rightarrow$

dispersion relation $\qquad \frac{p^2}{2m} = E = h\nu \qquad p = \frac{h}{\lambda} = \hbar k$

$h\nu = \frac{h^2}{\lambda^2}\frac{1}{2m} \qquad \hbar\omega = \frac{\hbar^2 k^2}{2m}$





monochromatic source

all crests arrive together

many harmonic components (travel at diff speeds

Lorentz force in terms of potentials: const. B

$$\Rightarrow \quad \vec{A} = -\frac{1}{2}(\vec{r}\times \underset{\uparrow\ \text{const}}{\vec{B}}) = -\frac{1}{2}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x & y & z \\ B_x & B_y & B_z \end{vmatrix} = -\frac{1}{2}(yB_z - zB_y)\,\hat{x} + \cdots$$

Canonical mom

$$\frac{d}{dt}(\underbrace{\vec{p}+q\vec{A}}_{P_{can}}) = -\vec{\nabla}\left(q\left[V - \vec{v}\cdot\vec{A}\right]\right)$$

$$= -\vec{\nabla}(q\underset{0}{V}) + \vec{\nabla}(\vec{v}\cdot\vec{A})$$

$$d\vec{P}_{can} = \vec{F} = q\underset{0}{\vec{E}} + q\vec{v}\times\vec{B}$$

$$\left[(\vec{v}\cdot\vec{\nabla})\vec{A}\right]_x = \underbrace{(\vec{v}\cdot\vec{\nabla})}A_x$$

cross product

$$\left(v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z}\right)\left(-\frac{1}{2}(yB_z - zB_y)\right) = v_y\left(-\frac{1}{2}\right)B_z + \frac{1}{2}v_zB_y$$

$$\frac{1}{2}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\frac{1}{2}\left(\vec{v} \times \vec{B}\right)$$