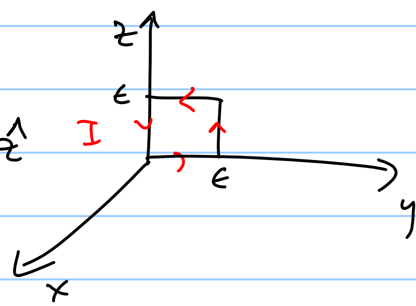
Lecture 40

Note Title 5/1/2006

Force on dipole $\vec{F} = \vec{\nabla}(\vec{m}\cdot\vec{B})$

$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$

$$\vec{B} = B_x(\,)\hat{x} + B_y(\,)\hat{y} + B_z(x,y,z)\hat{z}$$



$$d\vec{F} = I\, d\vec{l} \times \vec{B} = I\Big\{ \underbrace{dy\,\hat{y}\times\vec{B}(0,y,0)}_{\text{lower}} + \underbrace{dz\,\hat{z}\times\vec{B}(0,\epsilon,z)}_{\text{right side}} - \underbrace{dy\,\hat{y}\times\vec{B}(0,y,\epsilon)}_{\text{top}}$$

$$- dz\,\hat{z}\times\vec{B}(0,0,z) \Big\}$$

$$d\vec{F} = I\Big\{ -dy\,\hat{y}\times\Big[\overbrace{\vec{B}(0,y,\epsilon) - \vec{B}(0,y,0)}^{\epsilon \frac{\partial \vec{B}}{\partial z}}\Big] + dz\,\hat{z}\times\Big[\overbrace{\vec{B}(0,\epsilon,z) - \vec{B}(0,0,z)}^{\epsilon \frac{\partial \vec{B}}{\partial y}}\Big]\Big\}$$

Taylor series expansion of $\vec{B}(0,y,\epsilon) = \vec{B}(0,y,0) + \epsilon \left.\frac{\partial B}{\partial z}\right|_{0,y,0 \,=\, 0,0,0}$

$$\vec{F} = \int d\vec{F} \Rightarrow \int_0^{\epsilon} dy \frac{\partial \vec{B}}{\partial z} \simeq \epsilon \frac{\partial \vec{B}}{\partial z} \qquad 2\ \epsilon\text{'s}$$

$$\vec{F} = \underbrace{I\epsilon^2}_{m}\left[\hat{z}\times\frac{\partial\vec{B}}{\partial y} - \hat{y}\times\frac{\partial\vec{B}}{\partial z}\right] = m\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ 0 & 0 & 1\\ \frac{\partial B_x}{\partial y} & \frac{\partial B_y}{\partial y} & \frac{\partial B_z}{\partial y}\end{vmatrix} - m\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ 0 & 1 & 0\\ \frac{\partial B_x}{\partial z} & \frac{\partial B_y}{\partial z} & \frac{\partial B_z}{\partial z}\end{vmatrix}$$

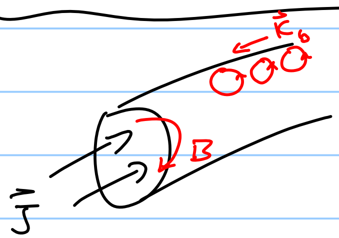
$$= m\left\{\hat{y}\frac{\partial B_x}{\partial y} \overbrace{- \hat{x}\frac{\partial B_y}{\partial y} - \hat{x}\frac{\partial B_z}{\partial z}}^{+\hat{x}\frac{\partial B_x}{\partial x}} + \hat{z}\frac{\partial B_x}{\partial z}\right\} = m\left\{\hat{x}\frac{\partial B_x}{\partial x} + \hat{y}\frac{\partial B_x}{\partial y} + \hat{z}\frac{\partial B_x}{\partial z}\right\}$$

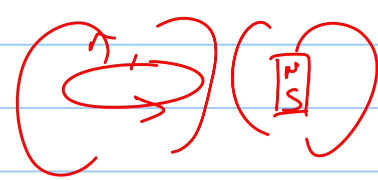
$$\vec{\nabla}\cdot\vec{B} = 0 = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = -\frac{\partial B_x}{\partial x}$$

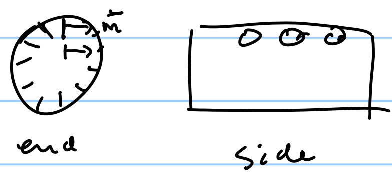
$$\vec{m} = m\hat{x} \qquad \vec{m}\cdot\vec{B} = mB_x \qquad \vec{\nabla}(m\cdot\vec{B}) = m\vec{\nabla}B_x = m\left\{\hat{x}\frac{\partial B_x}{\partial x} + \hat{y}\frac{\partial B_x}{\partial y} + \hat{z}\frac{\partial B_x}{\partial z}\right\}$$

$$m = I\epsilon^2$$

copper wire uniform $\vec{J} = J_0$





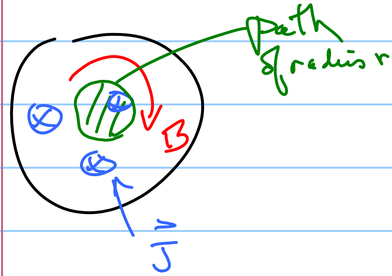


$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} = \mu_0$$

$$\frac{1}{\mu_0}\vec{\nabla}\times\vec{B} = \vec{J}_f + \underbrace{\vec{J}_b}_{\vec{\nabla}\times\vec{M}}$$

$$\vec{\nabla}\times\vec{H} = \vec{J}_f \qquad \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\int \vec{\nabla}\times\vec{H}\cdot d\vec{a} = \oint \vec{H}\cdot d\vec{l} = \qquad \vec{\nabla}\times\vec{H} = \vec{J}_f \cdot d\vec{a}$$



$$\int \vec{J}\cdot d\vec{a} = J\pi r^2 = I\frac{\pi r^2}{\pi R^2} = \oint \vec{H}\cdot d\vec{l} = 2\pi r H \qquad J = \frac{I}{\pi R^2} \qquad \oint |\vec{H}||d\vec{l}|\cos\theta$$

$$H = \begin{cases} I\frac{\pi r^2}{\pi R^2}\frac{1}{2\pi r} & \text{inside} \\ I\frac{1}{2\pi r} & \text{out} \end{cases}$$

$$\vec{B} = \mu H = \mu_0(1+\chi_m)\frac{Ir}{2\pi R^2} \quad \text{linear material}$$