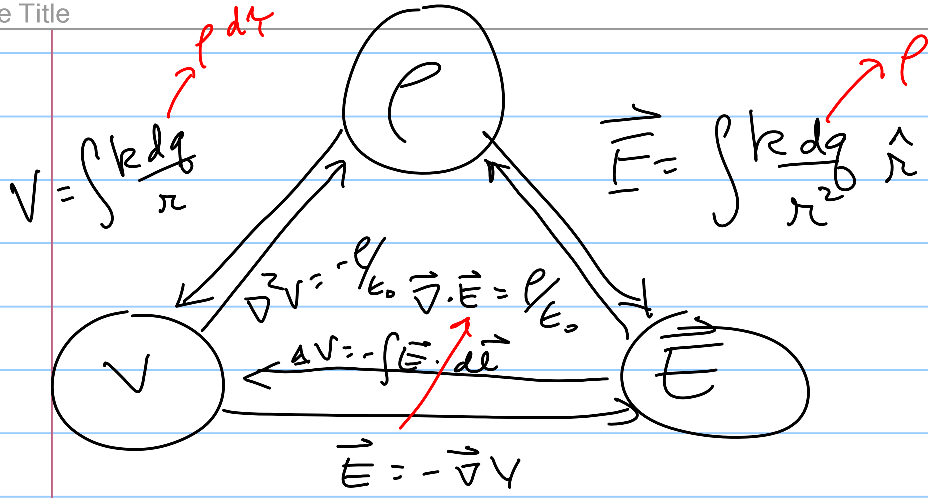
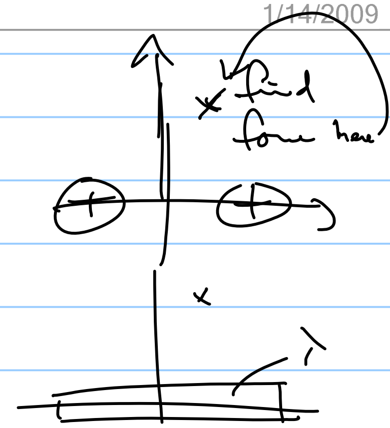
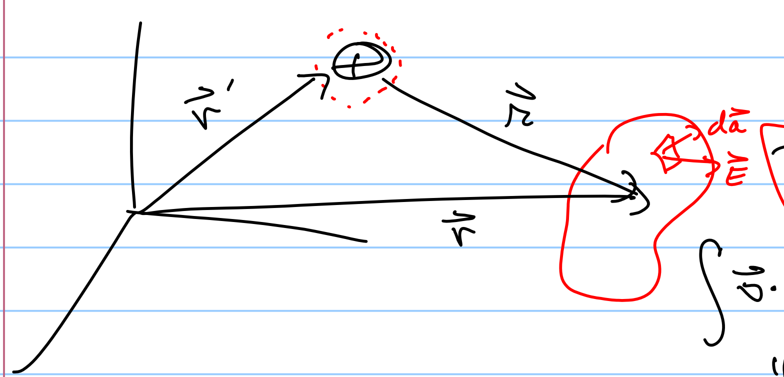
Note Title 1/14/2009





$$\vec{F} = q\vec{E}$$

Need V for energy methods



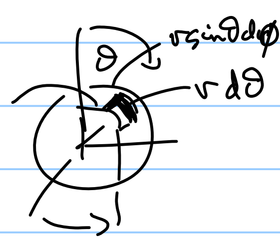
$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$ differential form

$\boxed{\text{DIVERGENCE TH.} \quad \int \vec{\nabla}\cdot\vec{E}\,d\tau = \oint \vec{E}\cdot d\vec{a}}$

$$\int \vec{\nabla}\cdot\vec{E}\,d\tau = \int \frac{\rho}{\epsilon_0}\,d\tau = \frac{Q_{enc}}{\epsilon_0}$$

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0} \quad \text{Integral form}$$

Put charge at origin $\quad \vec{E} = kq\frac{\hat{r}}{r^2} \quad \vec{r}\,' = 0$



$\vec{\nabla}\cdot\vec{E} \propto \vec{\nabla}\cdot\frac{\hat{r}}{r^2}$

DIVERGENCE THEOREM surface integral $r\sin\theta$

vol integral

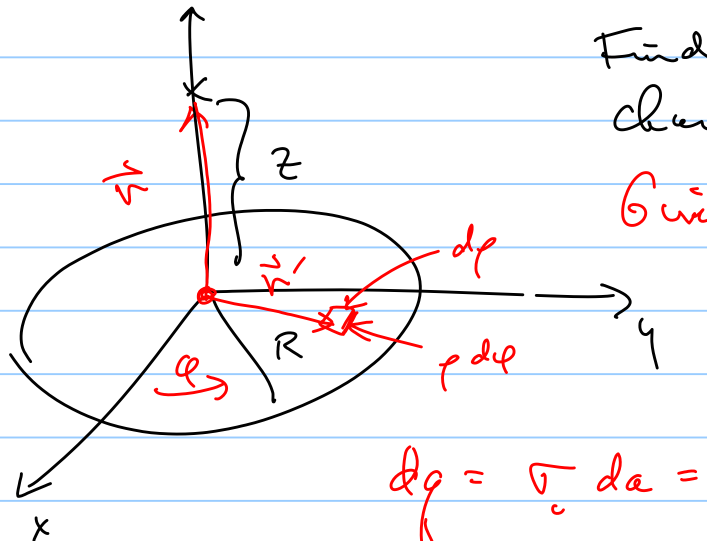
$$\underbrace{\int \vec{\nabla}\cdot\frac{\hat{r}}{r^2}\,d\tau}_{\delta^3(0) = \delta(x)\delta(y)\delta(z)} = \int \frac{\hat{r}}{\cancel{r^2}}\cdot \cancel{r^2}\sin\theta\, d\theta\, d\phi\, \hat{r} = \underline{4\pi}$$

$$\int \delta^3(r)\, d^3r = 1$$

encloses δ function

$$\vec{\nabla}\cdot\frac{\hat{r}}{r^2} = \delta^3(r)\,4\pi$$

$$\vec{\nabla}\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = 4\pi\,\delta^3(\vec{r}-\vec{r}\,')$$



Find $V(z)$ for a disc of charge with σ_0.

Given ρ find V

$$V = \int \frac{k\, dq}{\mathscr{r}}$$

$$dq = \sigma_0\, da = \sigma_0\, \rho\, d\rho\, d\varphi$$

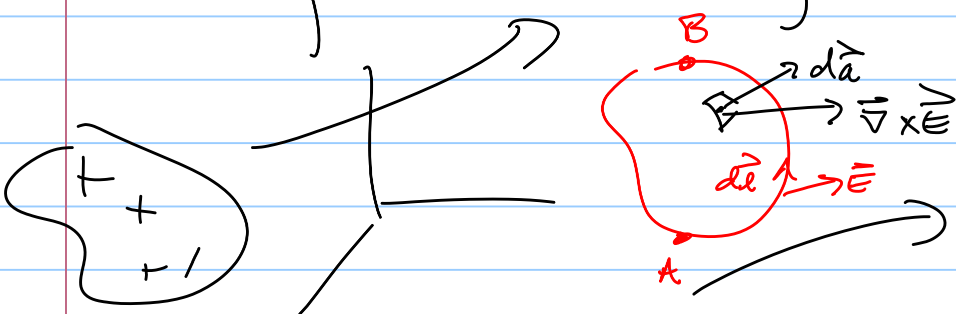
$$\vec{r} = z\hat{z}$$

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,' \qquad \vec{r}\,' = \rho\cos\varphi\,\hat{x} + \rho\sin\varphi\,\hat{y}$$

$$|\vec{\mathscr{r}}| = \sqrt{(0-\rho\cos\varphi)^2 + (0-\rho\sin\varphi)^2 + z^2}$$

$$V = \int_0^{2\pi}\!\!\int_0^{R} \frac{k\sigma_0\, \rho\, d\rho\, d\varphi}{\left[\rho^2\cos^2\varphi + \rho^2\sin^2\varphi + z^2\right]^{1/2}}$$

$$\int \vec{\nabla}\times\vec{E}\cdot d\vec{a} = \oint \vec{E}\cdot d\vec{l}$$

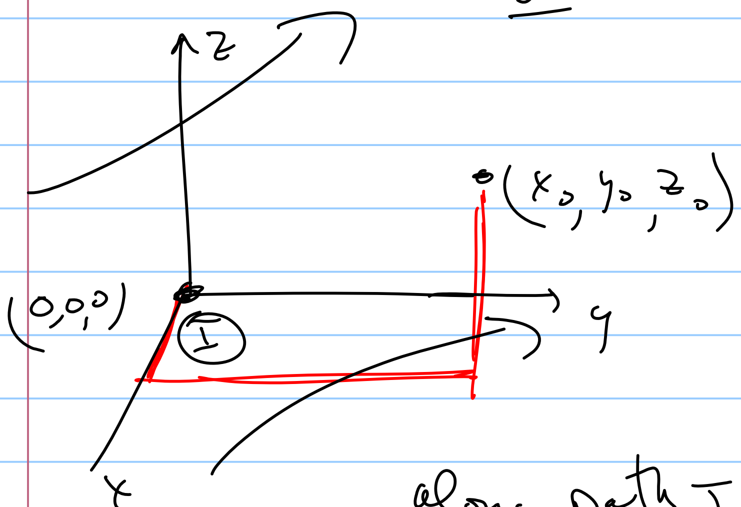


$$\vec{\nabla}\times\vec{E} = 0 \Rightarrow \oint \vec{E}\cdot d\vec{l} = 0$$

$$W = \int \vec{F}\cdot d\vec{l} = \int q\vec{E}\cdot d\vec{l} = q\int \vec{E}\cdot d\vec{l}$$

Problem: $\vec{E} = \underline{y^2}\,\hat{x} + (2xy + z^2)\,\hat{y} + 2yz\,\hat{z}$

$$d\vec{l} = \underline{dx}\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

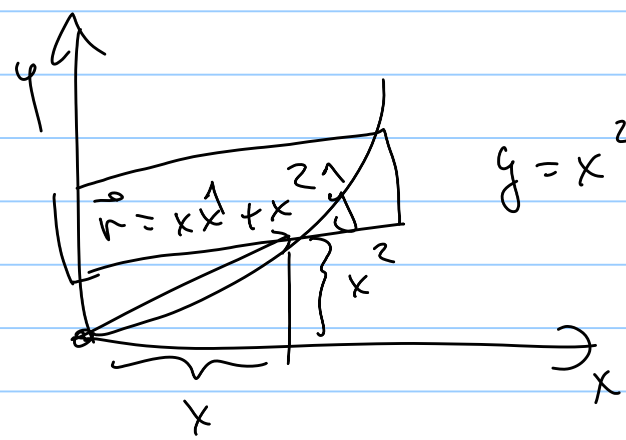


$$\vec{E}\cdot d\vec{l} = y^2dx + (2xy+z^2)dy + 2yz\,dz$$

along path I $\quad y = 0 \quad dy = 0$

$z = 0 \quad dz = 0$

go from $x = 0$ to $x = x_0$



$$y = x^2$$

$$d\vec{l} = dx\,\hat{x} + dy\,\hat{y}$$

$$y = x^2 \qquad dy = 2x\,dx$$

$$d\vec{l} = dx\,\hat{x} + 2x\,dx\,\hat{y}$$

$$d\vec{r} = d\vec{l} = dx\,\hat{x} + 2x\,dx\,y$$

$$v = \frac{d\vec{v}}{dt}$$