Wave intensity

$$\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{H} \qquad \text{for } \mu = 1 \quad \vec{H} = \vec{B}$$

want to calculate this from $\vec{E}$ field:

$$\vec{k} \times \vec{E} = \frac{\omega}{c} \vec{B}$$

$$\vec{S} = \frac{c}{4\pi} \frac{c}{\omega} \vec{E} \times (\vec{k} \times \vec{E})$$

ID: $\vec{A} \times (\vec{B} \times \vec{C}) = B(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

$$\vec{E} \times (\vec{k} \times \vec{E}) = \vec{k}(\vec{E} \cdot \vec{E}) - \vec{E}(\underbrace{\vec{E} \cdot \vec{k}}_{0})$$

$$\therefore \vec{S} = \frac{c^2}{4\pi} \frac{\vec{k}}{\omega} \vec{E} \cdot \vec{E}$$

in a medium $\omega = k_o c = (k_o n)\left(\frac{c}{n}\right)$

$$\vec{S} = \frac{c^2}{4\pi} \frac{k_o n}{k_o c} \hat{k} (\vec{E} \cdot \vec{E})$$

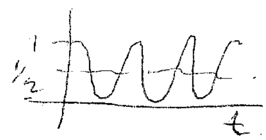
$$\vec{S} = \hat{k} \frac{nc}{4\pi} (\vec{E} \cdot \vec{E}) \qquad \text{direction of } \hat{k}$$

dot product is important to handle polarization states time average:

with real fields

$$\langle \vec{S} \rangle \propto \langle E_0^2 \cos^2 \omega t \rangle$$

$$\langle \cos^2 \omega t \rangle = \frac{1}{T}\int_0^T \cos^2 \omega t \, dt$$



$$= 1/2$$

either by setting $T = 2\pi/\omega$ or limit as $T \to \infty$

with complex fields

$$\langle \vec{S} \rangle = \frac{1}{2} \vec{E}^* \cdot \vec{E} \cdot \frac{nc}{4\pi} \hat{k}$$

this correctly eliminates $i\omega t$

see HM 5.3 for cautions about taking real part.

Alt expr:

$$\langle S \rangle = V_{ph} \langle \mathcal{E} \rangle \hat{k}$$

use caution when using these for

- non-monochromatic
- non-isotropic
- non linear

Polarization

given a direction of $\vec{k}$ $\quad \nabla \cdot \vec{E} = 0 \rightarrow \vec{k} \cdot \vec{E} = 0$
allows 2 degrees of freedom for $\vec{E}$ direction.

most general plane wave for $\vec{k} = k_0 \hat{z}$:
$$\vec{E}(\vec{r},t) = (E_x \hat{x} + E_y \hat{y}) e^{i(k_0 z - \omega t)}$$

amplitude coeff are complex:
$$E_x = E_{x0} e^{i\phi_x}$$
$$E_y = E_{y0} e^{i\phi_y}$$

$$\vec{E} = (E_{x0}\hat{x} + E_{y0} e^{i(\phi_y - \phi_x)} \hat{y}) e^{i(k_0 z - \omega t + \phi_x)}$$

relative phase $\phi_y - \phi_x$ determines nature of polarization state.

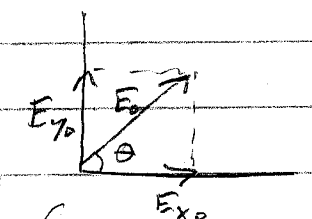
☆ an overall phase shift doesn't affect pol. state.

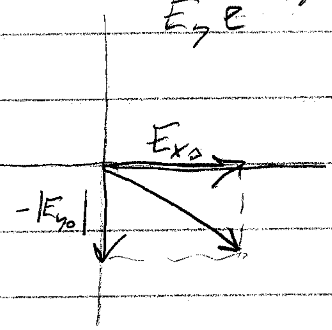
Linear $\quad \phi_x = \phi_y + n\pi \quad n = 0, 1, 2, \ldots$

$n = 0, 2, 4 \ldots$ $\qquad\qquad$ $n = 1, 3, 5 \ldots$

$E_y e^{i(\phi_y - \phi_x)} \rightarrow -|E_y|$

or more simply,
$\phi_x = \phi_y$
E_x, E_y can be $+/-$





$$\tan\theta = E_{y0}/E_{x0}$$

Jones vector $\begin{pmatrix} E_x \\ E_y \end{pmatrix}$ with E_x, E_y real or: $\vec{E} = E_0 \begin{pmatrix} a \\ b \end{pmatrix} e^{i(k_0 z - \omega t + \phi)}$

w/ $a^2 + b^2 = 1$ $\rightarrow a = \cos\theta \quad b = \sin\theta$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow$ horizontal $\qquad \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow$ vertical

Polarizer

a linear polarizer passes one direction of pol.

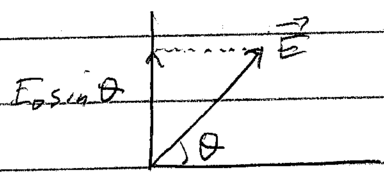
e.g. wire grid, polaroid, birefringent ...

what is transmission of polarizer as fcn. of angle?

$\vec{E} \rightarrow E_0 \begin{pmatrix} a \\ b \end{pmatrix}$

polarizer in vertical direction selects $E_0 \cdot b$ only.

$I(\theta) \propto E_0^2 \sin^2\theta$



$\vec{E}_{out} = \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}}_{\text{vertical pol.}} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ b \end{pmatrix}$

general case: polarizer oriented at angle α

rotate coord. system.

Circular polarization

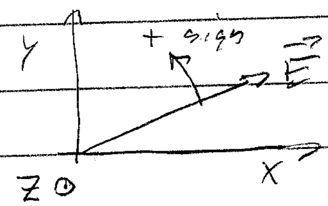
$$E_{x_0} = E_{y_0} \qquad \phi_x - \phi_y = \pm \frac{\pi}{2} \qquad (1/4 \text{ wave})$$

$$e^{\pm i \frac{\pi}{2}} = \pm i$$

$$\vec{E} = E_0 (\hat{x} \pm i \hat{y}) e^{i(k_0 z - \omega t)}$$

what is time dependence of $\vec{E}$ direction? choose $z=0$

$$Re(\vec{E}) \sim \hat{x} \cos \omega t \pm \hat{y} \sin \omega t$$



+ sign is called LH CP
("wrong sign")
convention was established very early

Jones vector

$$\vec{E} = E_0 \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm i \end{pmatrix} e^{i(k_0 z - \omega t)}$$

↳ normalization

Question:

a circ. polarized beam with intensity I_0 passes through a linear polarizer, oriented vertically.
What intensity gets through?

$$I_{out} \propto |E_{out}|^2 = |E_y|^2 = \left| \frac{1}{\sqrt{2}} i E_0 \right|^2 = \frac{1}{2} I$$

pick projection of $\vec{E}$ onto y axis

Now rotate polarizer → no change.
Projection of circle onto any axis is constant.

Elliptical polarization
- general case.

$$\vec{E} = E_0 (a\hat{x} + b\hat{y})\, e^{i(kz-\omega t)}$$

$$|a|^2 + |b|^2 = 1$$

only <u>relative</u> phase is impt
can keep <u>a</u> real if convenient.