Microwave Doppler Shift

In this lab, we want to build a classical Michelson interferometer, with a twist: one of the arms is mobile; the reflector is mounted on a car that moves down an air track, which has minimal friction. For our source, we will use microwaves, so we can expect a fringe spacing on the order of millimeters (what fringe spacing would we expect for optical frequencies?). The basic system configuration is shown in Figure 1.

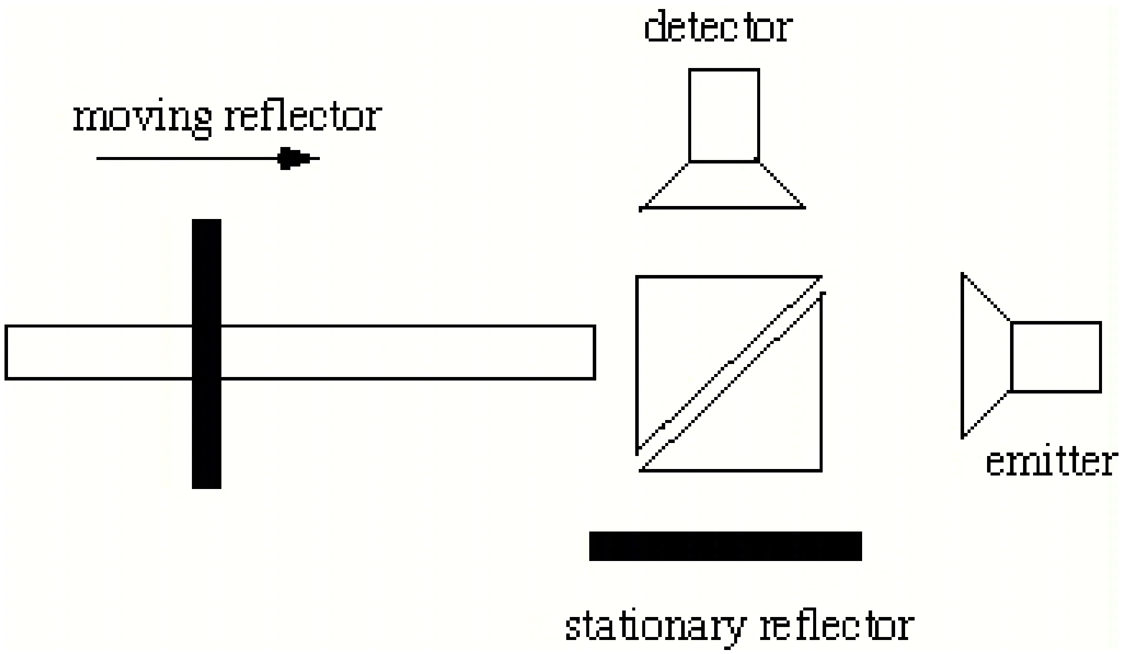

Figure 1. Schematic of a microwave Michelson interferometer with one moving reflector.

With this apparatus we can capture a fringe pattern in time. The detected field is the sum of the fields in the two arms. If we assume that the beamsplitter is 50/50, we can calculate the fringe pattern that we can expect to measure. The field entering each arm can be written as

$$0.5\ E_o \cos \omega t \tag{1}$$

or, in complex-exponential notation,

$$0.5 E_0 e^{i\omega t}\ . \tag{2}$$

If the path lengths $(l_2 - l_1)$ between the two arms give rise to a phase difference

$$\Delta = (2\pi/\lambda)\ \cdot 2(l_2 - l_1)\ , \tag{3}$$

we have at the detector

$$E_{\det}=0.5\,E_o e^{i\omega t}+0.5\,E_o e^{i(\omega t+\Delta)}\quad . \tag{4}$$

If we factor $0.5e^{i(\omega t+\Delta/2)}$ from both terms, we have

$$E_{\det}=0.5\,e^{i(\omega t+\Delta/2)}(e^{-i\Delta/2}+e^{i\Delta/2})=e^{i(\omega t+\Delta/2)}\cos(\Delta/2). \tag{5}$$

The detector follows only the average electric field; that is, it cannot detect the angular frequency ω of the microwave signal and instead averages the field over many cycles, so the measured field is

$$E_{\text{meas}}=\cos(\Delta/2)\quad . \tag{6}$$

Δ, however, is time dependent:

$$\Delta\to\Delta(t)\propto V\quad , \tag{7}$$

where *V* is the speed of the cart, so you can see fringes on the oscilloscope. The measured (average) field is a maximum every time $\Delta = 2\text{m}\pi$, where *m* is an integer (positive or negative). How can you measure velocity by counting fringes?

Suggestions:

1. Capture and evaluate a series of interference patterns for a variety of speeds and distances. Use these measurements to calculate the velocity of the source.

2. Simultaneously measure the cart speed using a stopwatch and ruler. Do the speeds measured using these two methods agree? What is the standard uncertainty of a measurement, presuming that you counted *N* fringes? Hint: The greatest probable error (100 % confidence interval) of the fringe count is *N*/2; calculate the resulting uncertainty of the velocity. What is the uncertainty of your clicking the stopwatch? Hint: It is not the same as your reflex time.

3. Alternate way to look at the interference pattern. Assume that the frequency of the beam in the moving arm is Doppler-shifted. (a) *Required*. Calculate the Doppler shift and show that you get the same frequency that you measured on the oscilloscope. It is incorrect to claim that the velocity of the cart is much less than the speed of light and therefore that the Doppler shift is irrelevant. The fringes you see on the oscilloscope can be interpreted as resulting from the Doppler shift. (b) Not required. Show that you can derive the equation $m\lambda = 2d$, which describes the positions of the interference maxima.
Hint:

$$E_{\det}=0.5\,E_o e^{i\omega t}+0.5\,E_o e^{i(\omega+\delta\omega)t}\quad , \tag{8}$$

where δω is the frequency shift due to the Doppler shift. Calculate δω and show that you get the same result as you get by using Eq. (6).