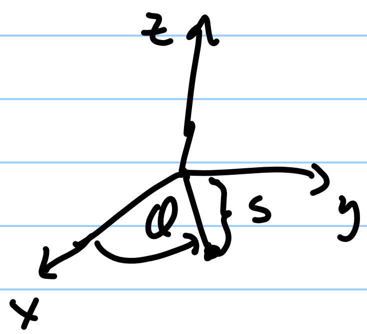
3.23 Laplace eqn in cylindrical coords



s is radial distance

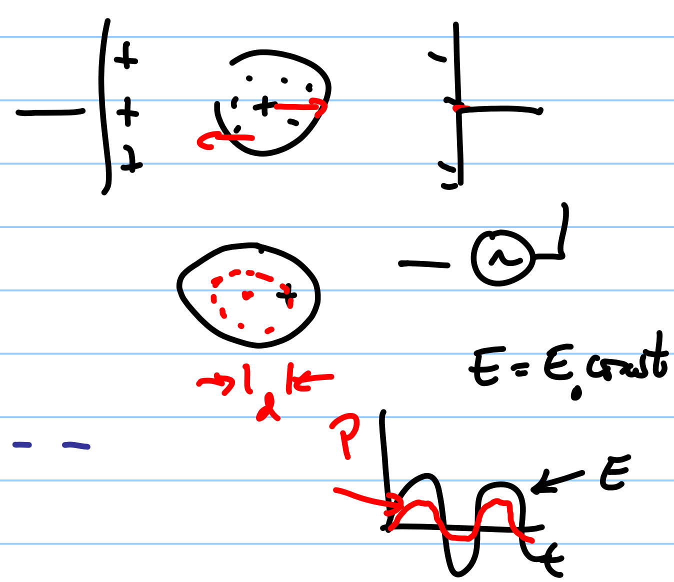
$$V = S(s)\,\Phi(\varphi)$$

$$s \frac{d}{ds}\left(s \frac{dS}{ds}\right) = k^2 S \qquad \text{where } C_2 = -k^2$$

$S = s^n$ works if $n = \pm k$

however case $k = 0$ has to be treated separately

Polarizability of atoms



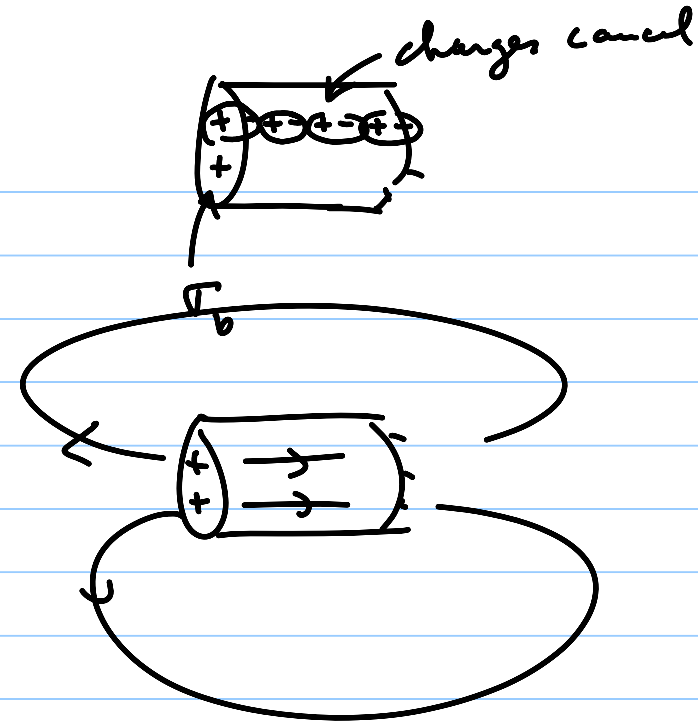
dipole mom qd

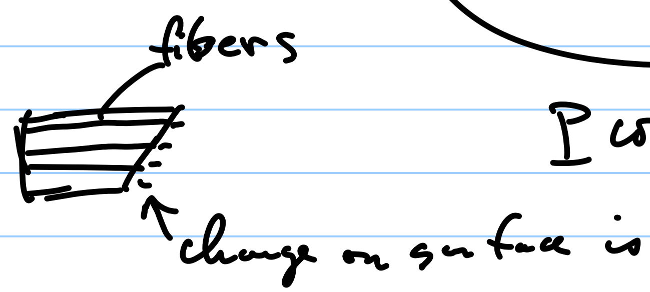
$p \propto E$

$$p = \alpha E + \beta E^2 + \cdots$$

polarizability

$\sigma_b = \vec{P}\cdot\hat{n}$



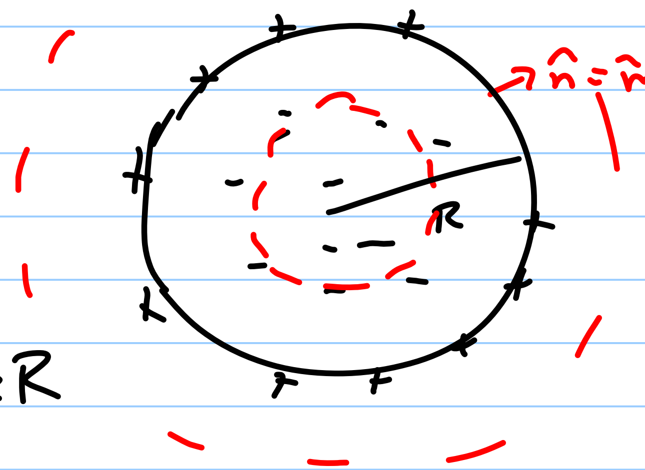


$P\cos\theta$ ← as θ increases σ decreases

$$\vec{\nabla}\cdot v = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 v_r + \cdots$$

InkSurvey question

$$\vec{P} = k\vec{r}$$



$$\sigma_b = \vec{P}\cdot\hat{n} = k\underset{\overset{\shortparallel}{r\hat{r}}}{\vec{r}}\cdot\hat{r}\Big|_{r=R} = kR$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k r\right) = -3k \quad \text{constant}$$

Gauss's

$$E_{in}\,4\pi r^2 = \frac{\int \rho\, d\tau}{\epsilon_0} = \frac{-3k\,\frac{4}{3}\pi r^3}{\epsilon_0}$$

$$\vec{E}_{in} = -\frac{k}{\epsilon_0} r\hat{r}$$

$$E_{out}\, 4\pi r^2 = \frac{Q_{encl}}{\epsilon_0} \Rightarrow Q_{tot} = \underbrace{\frac{4}{3}\pi R^3}_{vol}\big(\overset{\rho_{in}}{-3k}\big) + \underset{\sigma}{kR}\;\underset{area}{4\pi R^2}$$

$$Q_{tot} = 0 \qquad E_{out} = 0$$