Problem 3:

Note:

- The reference # for Eqns (4) + (6) are wrong. It is a cut + paste Error! Eqns (4) + (6) should Read

$$(4)\quad u_t = c^2 u_{xx}$$

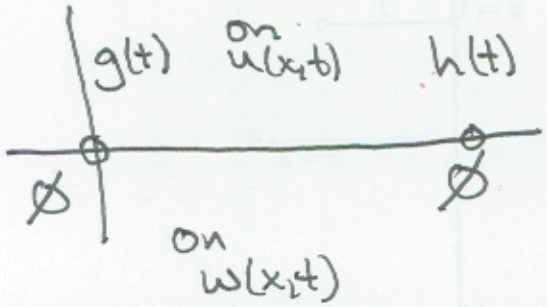
$$(6)\quad u(x,0) = f(x)$$

The idea is to let the background temp to be dynamic as given by

$$u(0,t) = g(t) \quad , \quad u(L,t) = h(t)$$

However, we never dealt with such boundary conditions. Our goal is to map the problem back to something understood.

Graphically we have



Q: What should w be?

A:

$$w(x,t) = \left(\quad ? \quad \right) - u(x,t)$$

where ? is some stuff that takes away $g(t)$ on the left and $h(t)$ on the right.

Guess

$$w(x,t) = \alpha x + \beta - u(x,t)$$

$$w(0,t) = \alpha \cdot 0 + \beta - u(0,t) = \beta - g(t) = 0 \Rightarrow \beta = g(t)$$

$$w(L,t) = \alpha \cdot L + \beta - u(L,t) = \alpha \cdot L + \beta - h(t) = 0$$

β: y intercept of $S(x,t)$

$$\Rightarrow \alpha = \frac{h(t) - g(t)}{L}$$

Slope of $S(x,t)$

$$u(x,t) = \alpha x + \beta - w(x,t)$$

$$u_t = (\alpha x + \beta)_t - w_t$$

$$u_{xx} = -w_{xx}$$

$$(4) \Leftrightarrow (\alpha x + \beta)_t - w_t = -c^2 w_{xx}$$

$$\Rightarrow w_t = c^2 w_{xx} + (\alpha x + \beta)_t$$

$$\Rightarrow S(x,t) = \alpha(t)x + \beta(t)$$

$$\alpha(t) = \frac{h(t) - g(t)}{L}, \quad \beta(t) = g(t)$$

with initial condition

$$w(x,0) = S(x,0) - f(x)$$