Hw ch3

Note Title 9/25/2007

Sec 4

9 $\vec{A} = 2\hat{i} + 3\hat{j}$ $\vec{B} = 4\hat{i} - 4\hat{j}$

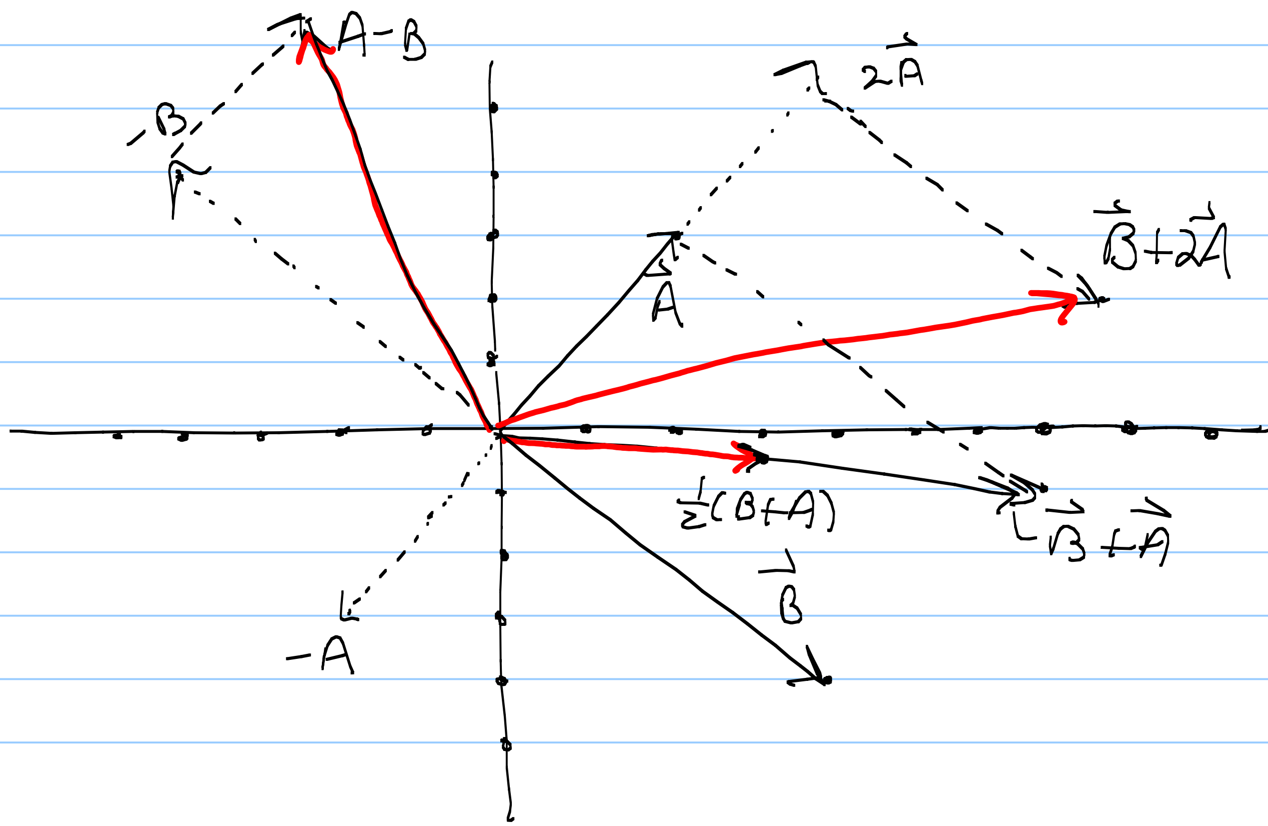
$-\vec{A} = -2\hat{i} - 3\hat{j}$ $\vec{A} - \vec{B} = -2\hat{i} + 7\hat{j}$

$3B = 12\hat{i} - 12\hat{j}$

$B + 2A = (4\hat{i} - 4\hat{j}) + (4\hat{i} + 6\hat{j})$

$= 8\hat{i} + 2\hat{j}$

$\frac{1}{2}(A + B) = \frac{1}{2}(6\hat{i} - \hat{j}) = 3\hat{i} - 6\hat{j}$



20) $\vec{a}_1 = \hat{i} + \hat{j}$ $\qquad \vec{a}_2 = \hat{i} - 2\hat{k}$

These are vectors in $\mathbb{R}^3$ i.e.,

$\vec{a}_1 = 1\cdot\hat{i} + 1\cdot\hat{j} + 0\cdot\hat{k}$

$\vec{a}_2 = 1\cdot\hat{i} + 0\cdot\hat{j} - 2\hat{k}$

Let $\vec{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$

$\vec{u}\cdot\vec{a}_1 = u_1 + u_2 = 0 \Rightarrow u_1 = -u_2$

$\vec{u}\cdot\vec{a}_2 = u_1 - 2u_3 = 0 \Rightarrow u_1 = 2u_3$

So $\vec{u} = \begin{bmatrix} u_1 \\ -u_1 \\ u_1/2 \end{bmatrix}$ for any u_1

i.e. $\vec{u} = \text{const.} \begin{bmatrix} 1 \\ -1 \\ 1/2 \end{bmatrix}$

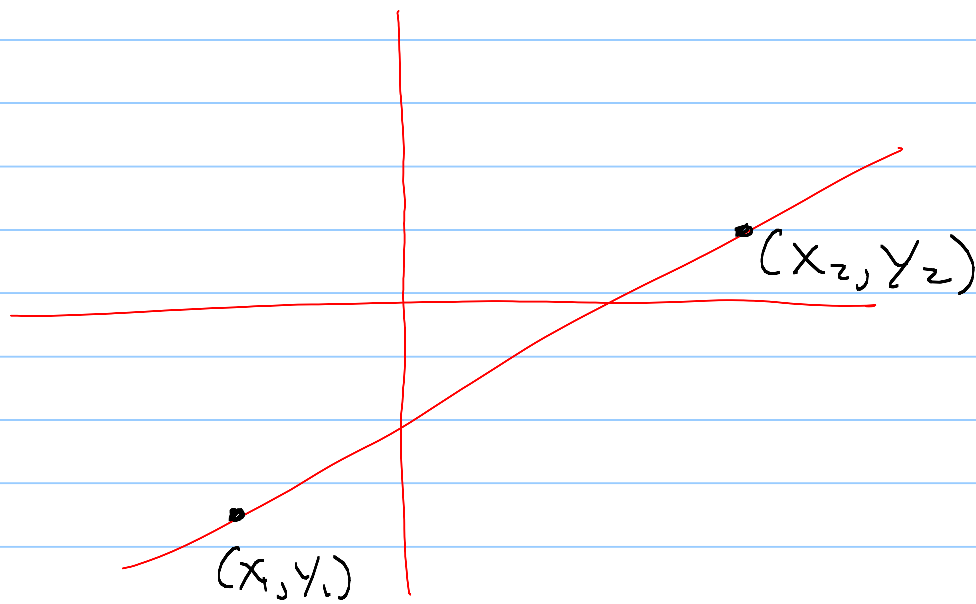
Note: we can also write

$\vec{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad \vec{a}_2 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$

$$\vec{u} \cdot \vec{a}_1 = [1, -1, 1/2] \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 1 - 1 = 0$$

$$\vec{u} \cdot \vec{a}_2 = [1, -1, 1/2] \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} = 1 - 1 = 0$$

Note Title 9/25/2007



A Line can be specified by any 2 points

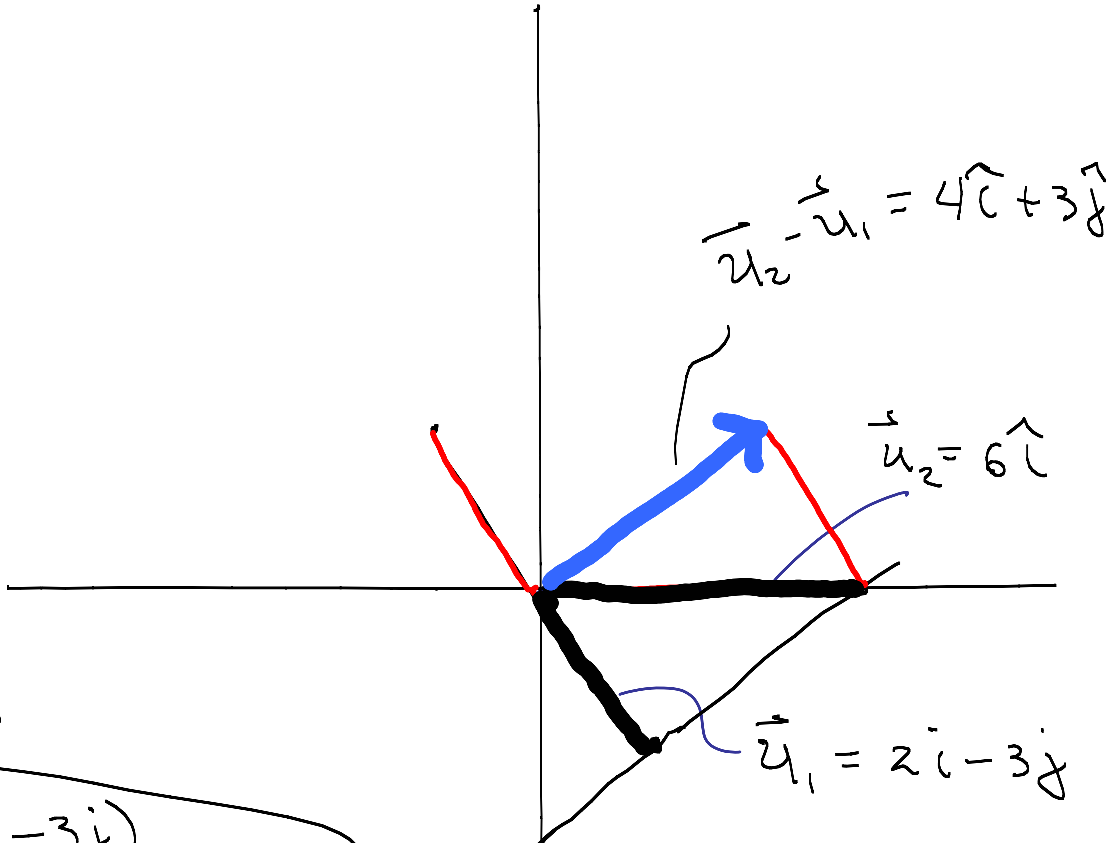
$$(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

Parametrically

$$\vec{r}(t) = \vec{r}_0 + \vec{A}t$$

i.e

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} A_x \\ A_y \end{pmatrix} t$$



so, eg

$$\boxed{\vec{r} = (2i - 3j) + (4i + 3j)t}$$

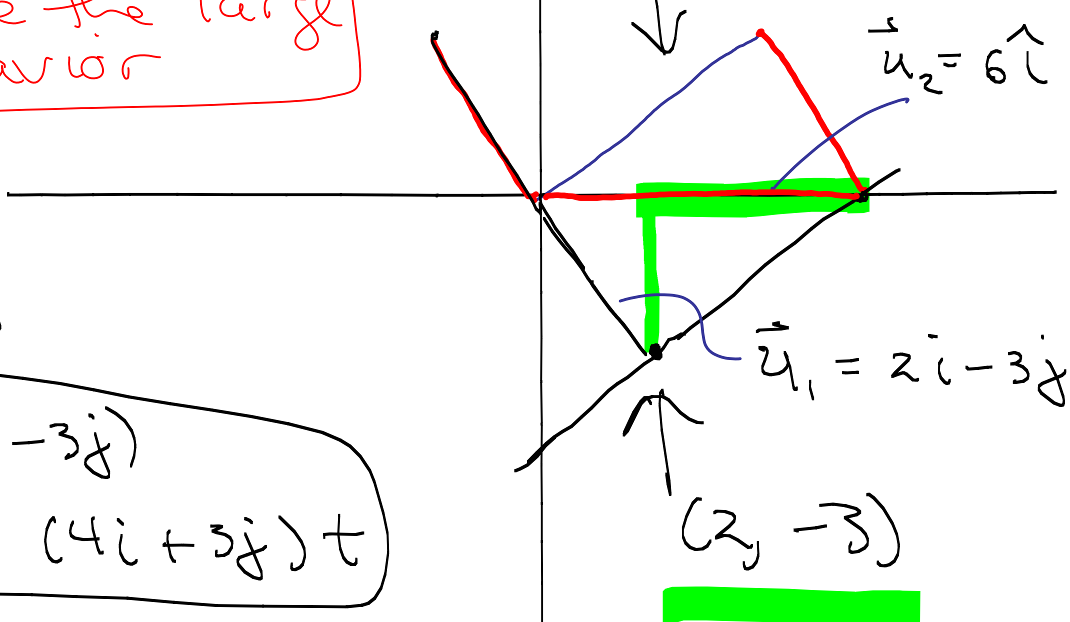
or

$$\boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \end{bmatrix} t}$$

3:5-1 Straight Line through $(2,-3)$ with slope $3/4$

$$\boxed{\vec{u}_2 - \vec{u}_1 = 4\hat{i} + 3\hat{j}}$$

must be the large t behavior



so, eg

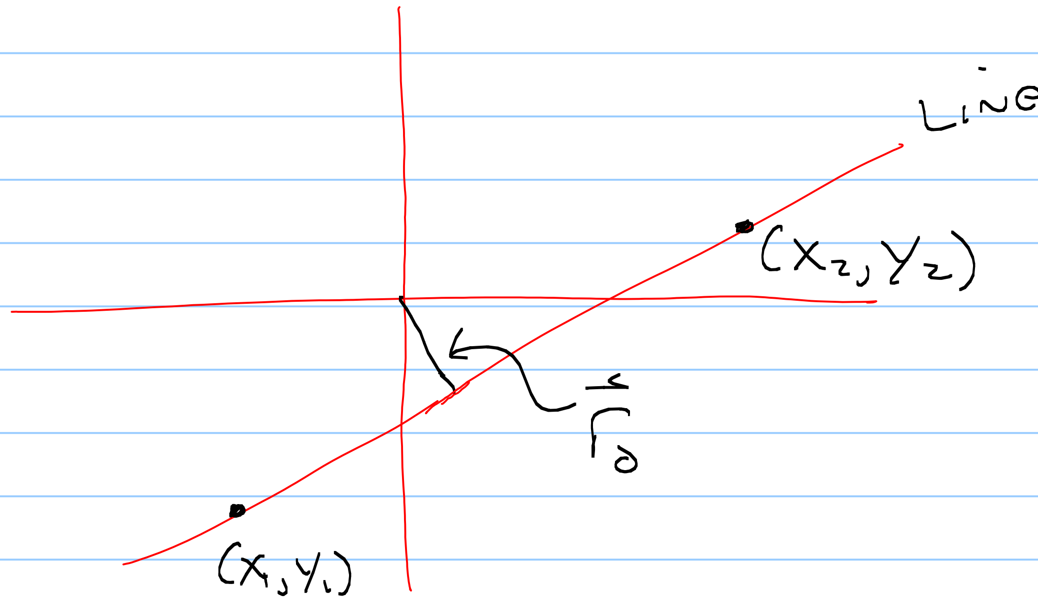
$$\boxed{\vec{r} = (2i - 3j) + (4i + 3j)t}$$

or

$$\boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \end{bmatrix} t}$$

could also use

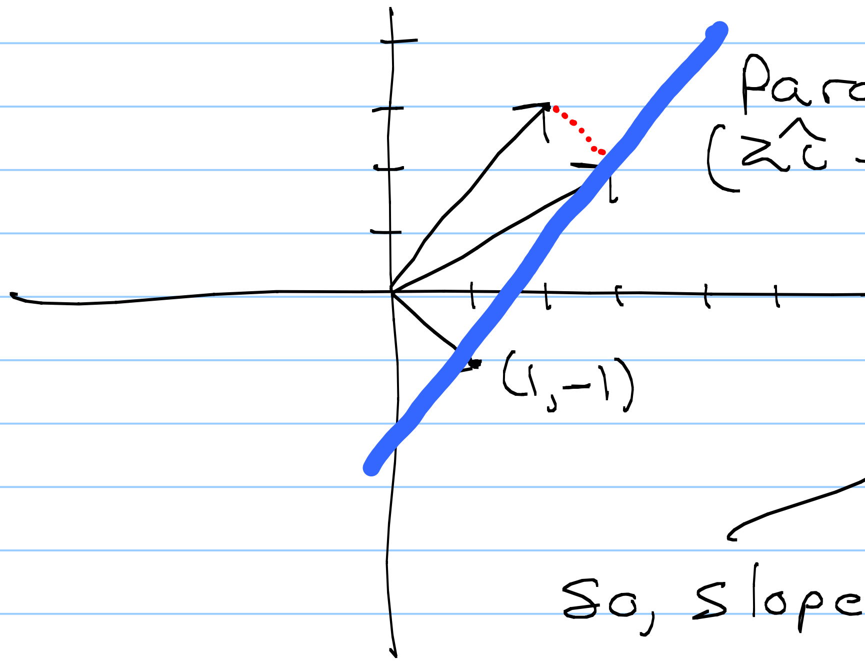
$$\boxed{\vec{r} = 6\hat{i} + (4\hat{i} + 3\hat{j})t}$$



slope of $\vec{r}_0 = -1/\text{slope of Line}$

3: 5-2

$$r = (\hat{i}-\hat{j}) + (2\hat{i}+3\hat{j})t$$



Parallel to $(2\hat{i}+3\hat{j})t$

So, slope of r is slope of $2\hat{i}+3\hat{j} = \frac{3}{2}$

21

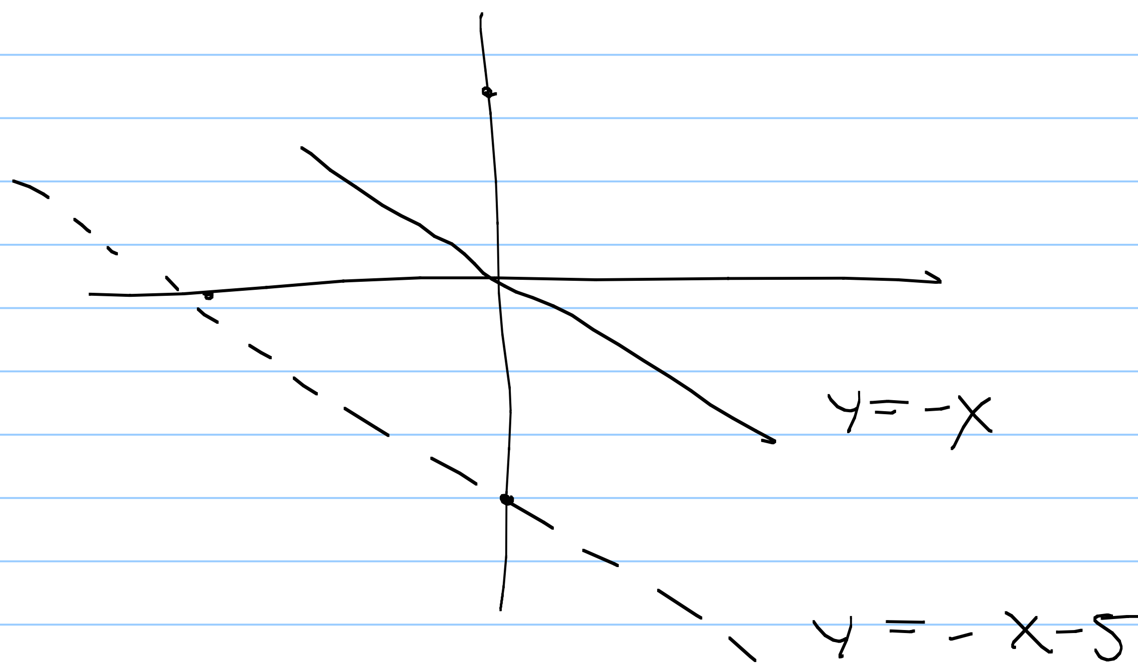
$$2x + 6y - 3z = 10$$

$$5x + 2y - z = 12$$

NB RHS does not affect slopes (angles) for instance

$x+y=0$ vs $y=-y$

$x+y=5$ $\quad$ $y=-x-5$



so it suffices to cons.

$$2x+6y-3z=0$$
$$5x+2y-z=0$$

These vectors are in the planes:

$$\begin{bmatrix} 2 \\ 6 \\ -3 \end{bmatrix} \qquad \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$$\vec{a}_1 \qquad \vec{a}_2$$

so $\cos\theta = \dfrac{\vec{a}_1 \cdot \vec{a}_2}{\|a_1\| \, \|a_2\|}$

$$\Rightarrow \cos\theta = \frac{10 + 12 + 3}{\sqrt{4+36+9}\,\sqrt{25+4+1}}$$

$$\boxed{\cos\theta = \frac{25}{7\sqrt{30}}}$$

37 $\quad \dfrac{x-1}{2} = \dfrac{y+3}{1} = \dfrac{z-4}{-3}$

This is equivalent to

$$\frac{x-1}{2} = y+3$$

$$\frac{x-1}{2} = \frac{z-4}{-3}$$

$$y+3 = \frac{z-4}{-3}$$

ie

$$x - 2y + 0z = 7$$

$$x + 0y + \tfrac{2}{3}z = \tfrac{8}{3} + 1 = {}^{11}\!/_{3}$$

$$0x + y + \tfrac{1}{3}z = \tfrac{4}{3} - 3 = -\tfrac{5}{3}$$

$$\begin{bmatrix} 1 & -2 & 0 \\ 1 & 0 & 2/3 \\ 0 & 1 & 1/3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 11/3 \\ -5/3 \end{bmatrix}$$

in order that  define a Line, the three equations cannot be simultaneously true [that would define a point.]

$Ax = Y$ has a unique solution $\iff$ Det $A \neq 0$.

Hence for our Line Det $A = 0$

This means that there are non zero vectors $\ni$ $A \cdot x = 0$

The set of all vectors such that

$A \cdot x = 0$ is the null space of A

To find $N(A)$ solve

$$Ax=0$$

For $A = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 0 & 2/3 \\ 0 & 1 & 1/3 \end{bmatrix}$

null space $= \left(-\frac{2}{3}, -1/3, 1\right) \equiv \vec{N}$

i.e. A dotted into this vector (or any multiple of it) is zero.

$$\vec{A} \cdot \alpha\vec{N} = \alpha A \cdot \vec{N} = 0$$

So $\vec{r} = r_0 + \vec{n}t$

$$\begin{bmatrix} 1 & -2 & 0 \\ 1 & 0 & 2/3 \\ 0 & 1 & 1/3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 11/3 \\ -5/3 \end{bmatrix}$$

Since there is a null space I can add $\alpha \vec{N}$ to any solution and get another solution

E.g. $\frac{1}{3}\begin{bmatrix} 11 \\ -5 \\ 0 \end{bmatrix} \equiv \vec{p}$

$$\underset{\text{particular}}{\underset{\uparrow}{\vec{p}}} + \alpha \underset{\text{null}}{\underset{\uparrow}{\vec{N}}}$$

$$\boxed{\vec{r} = \frac{1}{3}\begin{bmatrix} 11 \\ -5 \\ 0 \end{bmatrix} + \begin{bmatrix} -2/3 \\ -1/3 \\ 1 \end{bmatrix} t}$$

By the same argument

$$\frac{x+3}{4} = \frac{y+4}{1} = \frac{8-z}{4}$$

$$\Rightarrow \begin{bmatrix} 1 & -4 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1/4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 5 \\ -2 \end{bmatrix}$$

$$\vec{r} = \underbrace{\begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}}_{\text{part.}} + \underbrace{\begin{bmatrix} -1 \\ -1/4 \\ 1 \end{bmatrix}}_{\text{null space}} t$$

For large t the two lines are charac. by the vectors

$$\begin{bmatrix} -1 \\ -1/4 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} -2/3 \\ -1/3 \\ 1 \end{bmatrix}$$

Dot prod.

$$\frac{2}{3} + \frac{1}{12} + 2 \neq 0$$

So they must intersect.

$$\vec{r} = \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ -1/4 \\ 1 \end{bmatrix} t$$

$$\vec{r} = \frac{1}{3}\begin{bmatrix} 11 \\ -5 \\ 0 \end{bmatrix} + \begin{bmatrix} -2/3 \\ -1/3 \\ 1 \end{bmatrix} t$$

$$0 = \begin{bmatrix} 11/3 - 5 \\ -5/3 + 2 \\ 0 \end{bmatrix} + \begin{bmatrix} -2/3 + 1 \\ -1/3 + 1/4 \\ 0 \end{bmatrix} t$$

$$0 = \begin{bmatrix} -4/3 \\ 1/3 \\ 0 \end{bmatrix} + \begin{bmatrix} 1/3 \\ -1/2 \\ 0 \end{bmatrix} t$$

$$\begin{pmatrix} \frac{4}{3} \\ -1/3 \\ 0 \end{pmatrix} = t \begin{pmatrix} 1/3 \\ -1/2 \\ 0 \end{pmatrix} \Longleftrightarrow t = 4$$