Note Title 11/13/2006

11/13/06

Chladni figures demo

Degeneracy

Eigenfrequencies
Normal mode frequencies

we showed last time that

$$\omega_{n,m}^2 = c^2 \pi^2 \left(\frac{n^2}{L_x^2} + \frac{m^2}{L_y^2} \right)$$

So if the drum is square, then $L_x = L_y \equiv L$ and

$$\omega_{n,m}^2 = \left(\frac{c\pi}{L} \right)^2 (n^2 + m^2)$$

this says that

$$\omega^2_{n,m} = \omega^2_{m,n}$$

i.e. 2 modes have the same frequency:

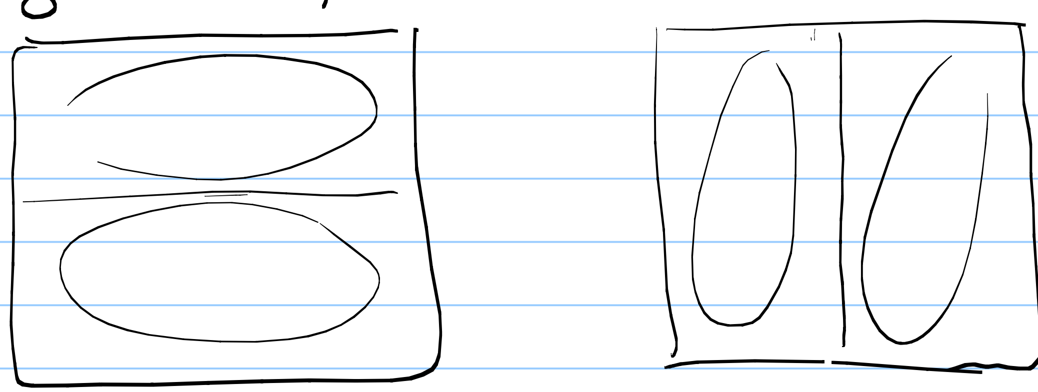
n, m mode =

$$\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi y}{L}\right)$$

m, n mode =

$$\sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi y}{L}\right)$$

E.g. 1, 2 2, 1



what happens when 2 modes compete?

rather than continue on to ∇^2 in 3 cartesian coordinates, lets jump to spherical coordinates.

See wiki link to mathworld for derivation

Laplacian in spherical coordinates.

$$\nabla^2 \psi(x,y,z) = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}$$

$$\nabla^2 \psi(r,\theta,\varphi) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \psi}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 \psi}{\partial \varphi^2}$$

$\nabla^2 \psi = 0$ Laplace Equation

As usual we suppose

$$\psi(r,\theta,\varphi) = R(r)P(\theta)\,\Phi(\varphi) \Rightarrow$$

$$\nabla^2 \psi = \frac{1}{r^2} P\Phi \frac{d}{dr}\left(r^2 \frac{dR}{dr}\right)$$

$$+ \frac{R\Phi}{r^2 \sin\theta} \frac{d}{d\theta}\left(\sin\theta \frac{dP}{d\theta}\right)$$

$$\frac{RP}{r^2 \sin^2\theta} \frac{d^2\Phi}{d\varphi^2} = 0$$

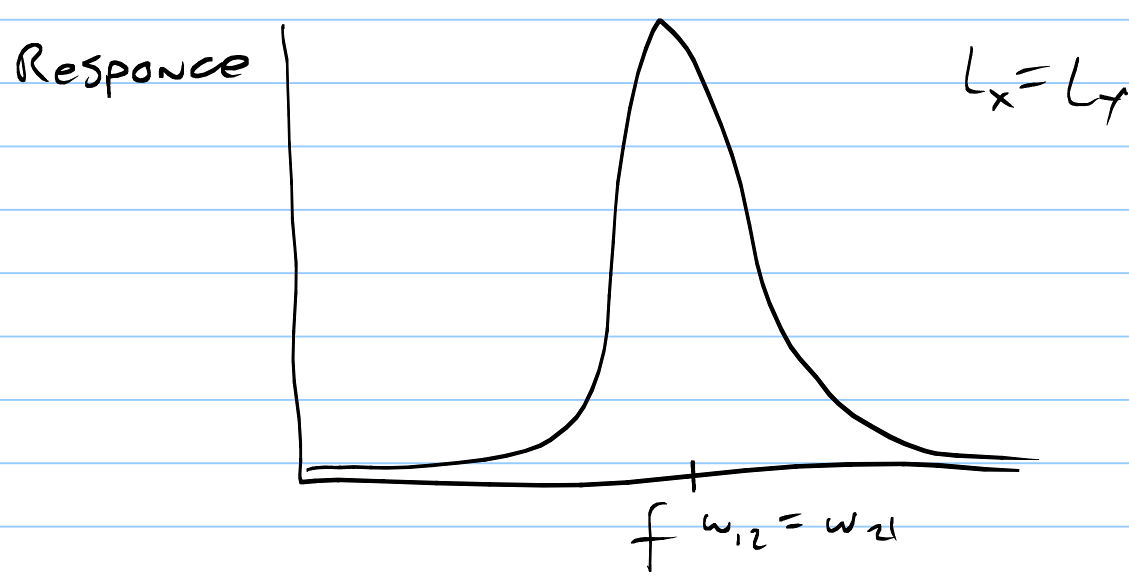
Divide by $RP\Phi = \psi$

$$\frac{1}{r^2 R} \frac{d}{dr}\left(r^2 R'\right) + \frac{1}{P r^2 \sin\theta} \frac{d}{d\theta}\left(\sin\theta P'\right)$$

$$+ \frac{1}{\Phi r^2 \sin^2\theta} \Phi'' = 0$$

multiply by $r^2 \sin^2\theta$

$$\frac{\sin^2\theta}{R}\frac{d}{dr}\left(r^2 R'\right) + \frac{\sin\theta}{P}\frac{d}{d\theta}\left(\sin\theta P'\right) = -\frac{\Phi''}{\Phi}$$



$L_x \neq L_y$ $\quad$ $L_x \approx L_y$