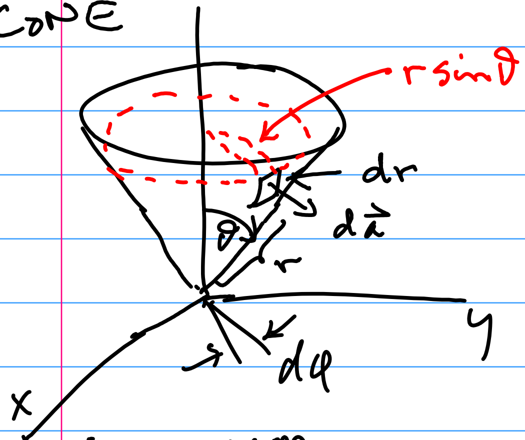
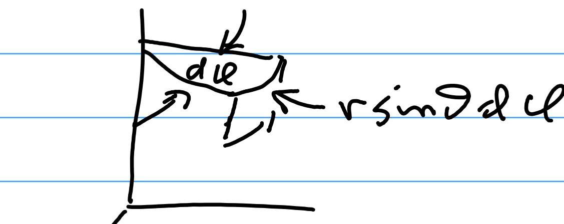
Lecture 9   Shadowitz parts of sections 1-5, 1-6, 1-7, and 3-3 may be covered.

InkSurvey question for Monday

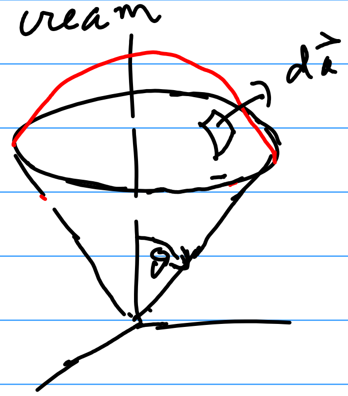
CONE





$$d\vec{a} = r\sin\theta\, d\varphi\, dr\ \hat{\theta}$$

ice cream



$$d\vec{a} = r\sin\theta\, d\varphi\, r\, d\theta\ \hat{r}$$

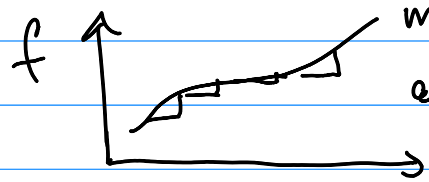
Using the spherical unit vectors is useful when calculating the flux given E in spherical coordinates. Then you have E dot da where the unit vectors go away due to the dot product.

InkSurvey question for Monday

Fundamental theorem of calculus

$$\int_a^b \frac{df}{dx}\,dx = \int_a^b df = f(b) - f(a)$$

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$



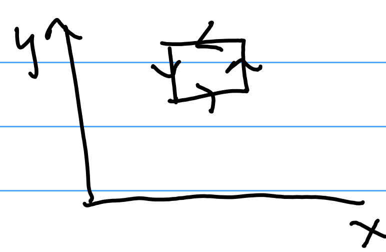
multiply $\frac{\Delta f}{\Delta x}$ by Δx and sum

Defn of curl:

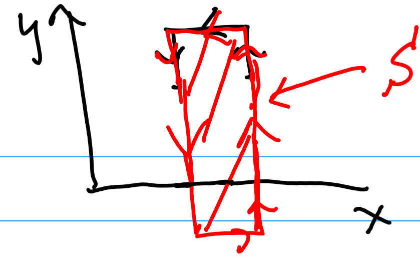
$$\left(\vec{\nabla}\times\vec{F}\right)_z \equiv \lim_{\Delta S_z \to 0}\left[\frac{1}{\Delta S_z}\oint_{C_z} \vec{F}\cdot d\vec{r}\right]$$

multiply both sides by ΔS_z

$$\vec{\nabla}\times\vec{F}\cdot\Delta\vec{S} = \oint_{\text{around } \Delta S} \vec{F}\cdot d\vec{r}$$



$$\sum_{S'} \vec{\nabla}\times\vec{F}\cdot\Delta\vec{s} = \oint_{\text{around } S'} \vec{F}\cdot d\vec{r}$$



$$\sum \to \int \Rightarrow \text{Stokes theorem} \quad \int \vec{\nabla}\times\vec{F}\cdot d\vec{a} = \oint \vec{F}\cdot d\vec{r}$$

$d\vec{l}$ "

sum inside bndry

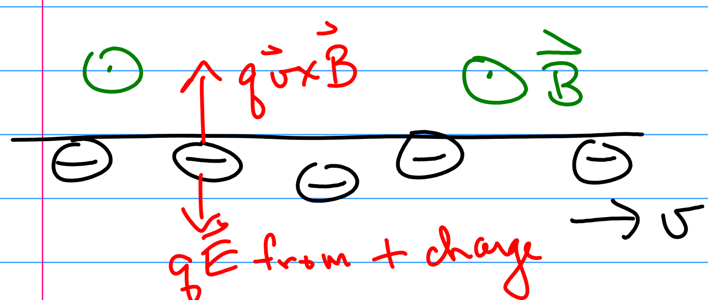
boundary integral

Fundamental th. cal says sum of df's from x=a to x=b is the difference in f(b)-f(a), that is the values of the function at the boundary.

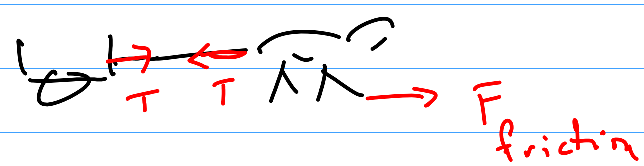
For curls the sum is over the interior of an area and the result is a sum (a line integral) over the boundary.

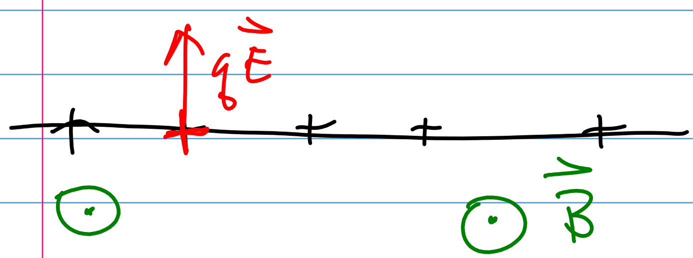
InkSurvey question from Friday

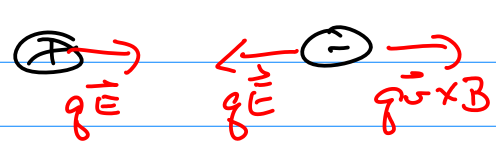
What is the net force on a positive charge?



This is like the horse pulling the cart.



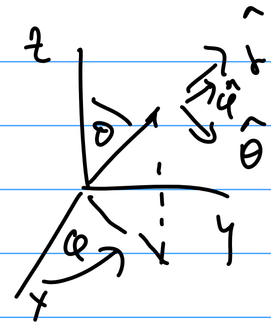




Muddiest points:

(1) How was the Lorentz force determined?

By measuring forces on current carrying wires.



(2) Cross product in spherical coords.

$\hat{\theta} \times \hat{\varphi} = \hat{r}$ $\quad \hat{\varphi} \times \hat{r} = \hat{\theta}$

(3) Interpretation of curl on the surface of water.

Put a paddle wheel horizontally just under the water surface so it generates a vortex.

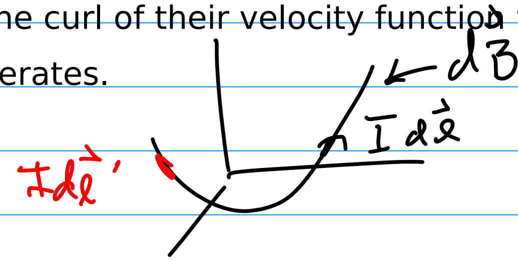
Pine needles just above will move in a circle. The curl of their velocity function will be non-zero

(4) Force of wire on itself due to the B it generates.

$I d\vec{\ell}\,'$ generates $d\vec{B}$ at $I d\vec{\ell} \Rightarrow$ Force

$I d\vec{\ell}$ generates dB at $I d\vec{\ell}\,' \Rightarrow$ opposite Force

The net force on wire = 0



(5) What's the point of the curl?

We will use it to model magnetic fields from currents.

(6) So much material. How do I study for the next exam?

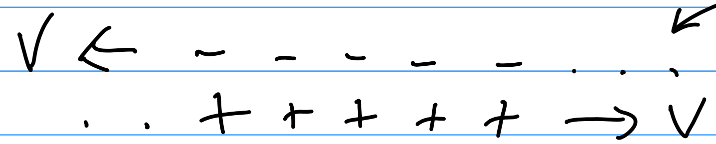
I will talk about what will be on the exam.

(7) Relativity and electric and magnetic fields.

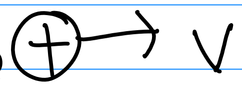
See below

(2) Fix charges so v = 0 then F = q E + 0

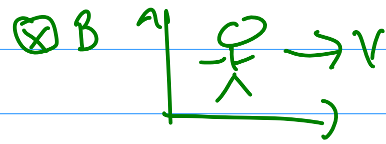
Infinite lines of charge



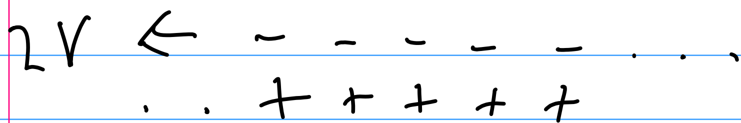
$I \rightarrow 2\lambda V$

Q 

What is the force on Q?



Go to a reference frame moving to the right at speed V



Q 

What is the force on Q?

There is NO $q\vec{v}\times\vec{B}$ since $v = 0$

$$\vec{F} = q\vec{E} + \underbrace{q\vec{v}\times\vec{B}}_{0 \text{ since } v=0}$$

so $E \neq 0$

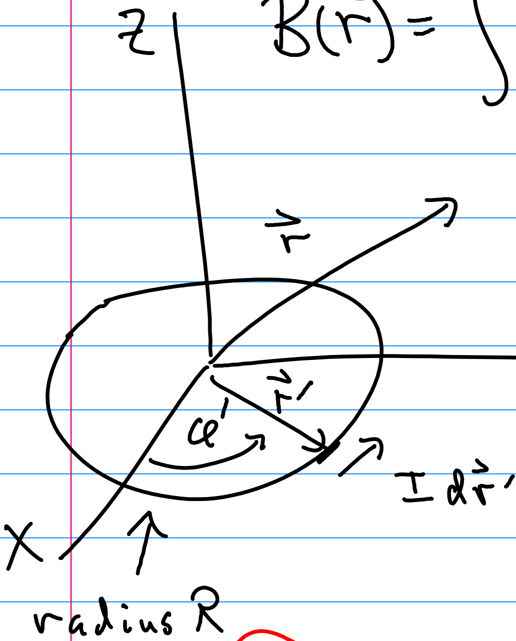
Moving charge density is greater due to length contraction so the negative charge density is greater than the positive thus attracting Q as expected.

We will continue with a mathematica analalysis once we know B from an infinite current moving along a line.

What have we covered?

Calculate B given currents Law of Biot + Savart

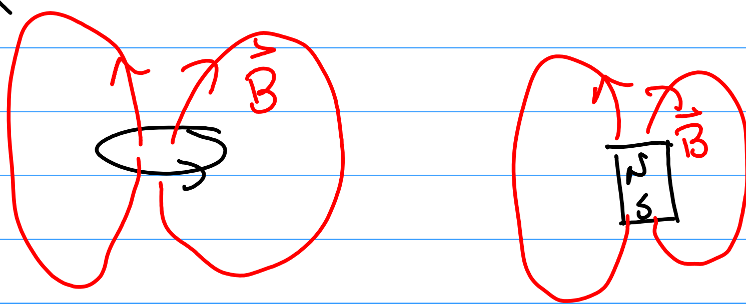
$$\vec{B}(\vec{r}) = \int d\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{r}\,' \times (\vec{r}-\vec{r}\,')}{(\vec{r}-\vec{r}\,')^3}$$



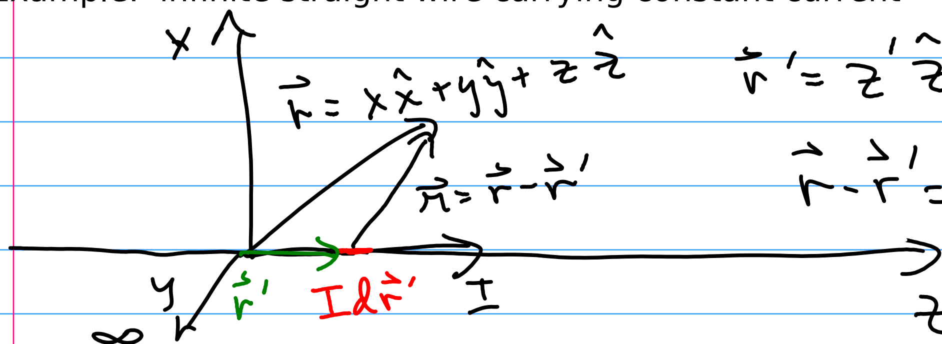
$$\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$$

$$d\vec{r}\,' = R\,d\varphi'(-\sin\varphi'\,\hat{x} + \cos\varphi'\,\hat{y})$$

$$\vec{r}\,' = R\cos\varphi'\,\hat{x} + R\sin\varphi'\,\hat{y}$$



Example: infinite straight wire carrying constant current



$$\vec{r}\,' = z'\,\hat{z} \qquad d\vec{r}\,' = dz'\,\hat{z}$$

$$\vec{r} - \vec{r}\,' = x\hat{x} + y\hat{y} + (z - z')\hat{z}$$

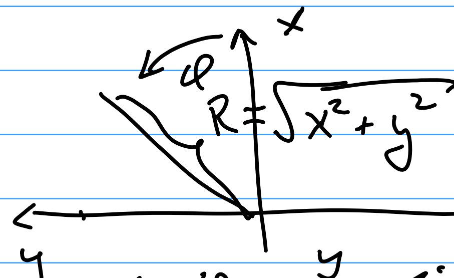
$$\vec{B} = \frac{\mu_0}{4\pi}\int_{-\infty}^{\infty} \frac{I\,d\vec{r}\,' \times (\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} \qquad \vdots \qquad \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3} = \frac{x\hat{x} + y\hat{y} + (z-z')\hat{z}}{\left(x^2 + y^2 + (z-z')^2\right)^{3/2}}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & dz' \\ \frac{x}{(\ )^{3/2}} & \frac{y}{(\ )^{3/2}} & \frac{z-z'}{(\ )^{3/2}} \end{vmatrix}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \left[\hat{x}\left(\frac{-dz' y}{(\ \ )^{3/2}} \right) + \hat{y} \frac{dz' x}{(\ \ )^{3/2}} \right]$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \hat{x} \int_{-\infty}^{\infty} \frac{-dz' y}{\left(x^2+y^2+(z-z')^2\right)^{3/2}} + \frac{\mu_0 I}{4\pi} \hat{y} \int_{-\infty}^{\infty} \frac{dz' x}{\left(x^2+y^2+(z-z')^2\right)^{3/2}}$$

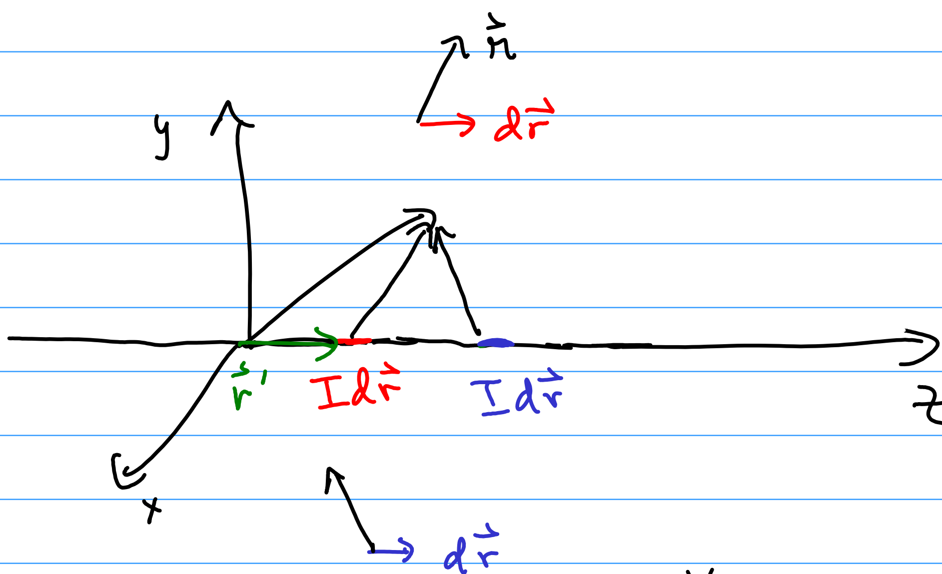
$$\vec{B} = \frac{\mu_0 I}{4\pi} \left(\frac{-2y}{x^2+y^2} \hat{x} + \frac{2x}{x^2+y^2} \hat{y} \right)$$



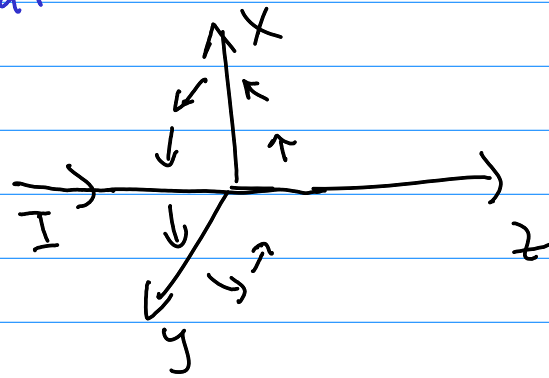
$$\vec{B} = \frac{\mu_0}{2\pi} \left(\frac{1}{R} \sin\varphi \, \hat{x} - \frac{1}{R} \cos\varphi \, \hat{y} \right)$$

$\cos\varphi = \frac{y}{R}$ $\sin\varphi = \frac{x}{R}$

$$\vec{B} = \frac{\mu_0}{2\pi R} \hat{\varphi}$$



$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\varphi}$$



What are we going to cover?

Helmholtz theorem says we need $\vec{\nabla}\cdot$ & $\vec{\nabla}\times$ vector function to uniquely determine it.

General result

$$\vec{B}(x,y,z) = \int d\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')\times(\vec{r}-\vec{r}\,')}{(\vec{r}-\vec{r}\,')^3}\,dx'dy'dz'$$

What is $\vec{\nabla}\cdot\vec{B}$?

$$\vec{\nabla}\cdot\frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')\times(\vec{r}-\vec{r}\,')}{(\vec{r}-\vec{r}\,')^3}\,dx'dy'dz'$$

$$= \frac{\mu_0}{4\pi}\int \vec{\nabla}\cdot\left[\frac{\vec{J}(\vec{r}\,')\times(\vec{r}-\vec{r}\,')}{(\vec{r}-\vec{r}\,')^3}\right] dx'dy'dz'$$

$$\vec{\nabla}\cdot\left(\frac{\vec{J}\times\hat{r}}{r^2}\right) = \frac{\hat{r}}{r^2}\cdot(\vec{\nabla}\times\vec{J}) - \vec{J}\cdot\left(\vec{\nabla}\times\frac{\hat{r}}{r^2}\right)$$

Questions?

$$\vec{\nabla}\times\vec{J} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ J_x(x',y',z') & J_y(x',y',z') & J_z(x',y',z') \end{vmatrix} = 0$$

No x,y,z dependence

$\vec{\nabla} \times \vec{J} =$