Lecture 25 Shadowitz Chapter 15.1

Maxwell's eqns in vaccum:

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = \mu_0 \underset{\substack{\parallel \\ 0 \text{ vacuum}}}{\vec{J}} + \underbrace{\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}}_{\vec{J}_{\text{displacement}}}$$

$\vec{J} = \rho \vec{v}$

$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0 \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$\rho = 0$ vacuum

These are coupled partial differential equations.

in vacuum

For hmwk you are to show that these can be uncoupled by

(a) $$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) \xrightarrow{\text{show}} \nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

(b) $$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) \xrightarrow{\text{show}} \nabla^2 \vec{B} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

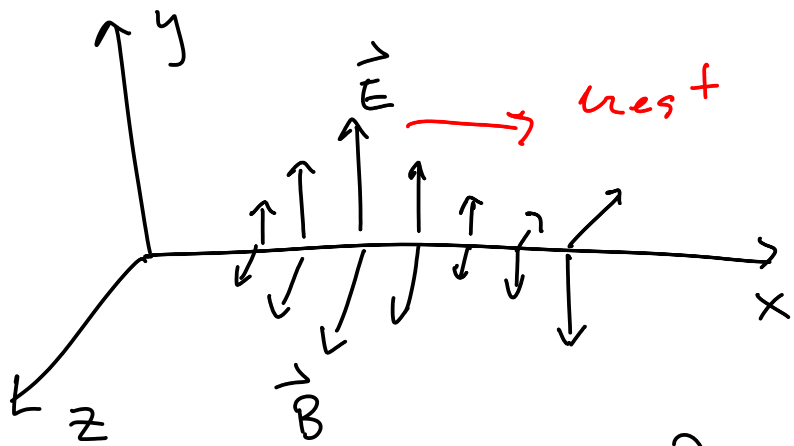
vector Laplacian

$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ is the speed of a wavecrest

$c \simeq 3 \times 10^8$ m/s

Example: I will give the soln for a plane electromagnetic wave in vacuum

We will then show (lecture and hmwk) how this satisfies M.E.'s



$$\vec{E} = E_0 \hat{y} e^{i(kx-\omega t)}$$

$$\vec{B} = B_0 \hat{z} e^{i(kx-\omega t)}$$

take real part of any soln

Traveling waves which are in phase.

$$\lambda \nu = c \qquad \frac{2\pi\nu}{2\pi/\lambda} = \frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

How does this soln satisfy the differential form of ME's?

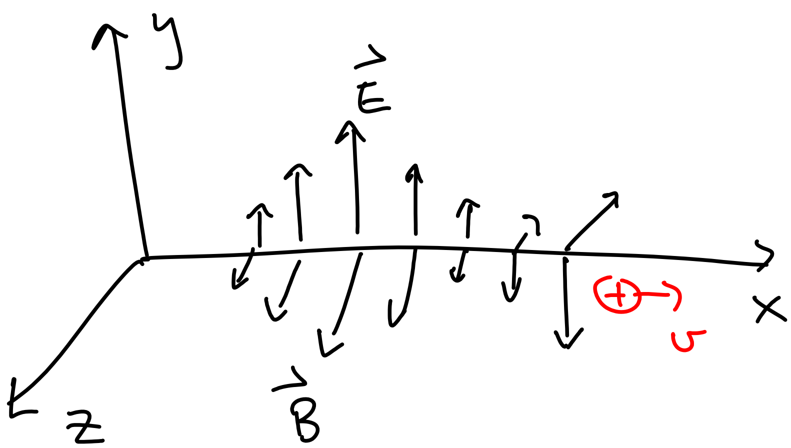
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} : \quad \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_0 e^{i()} & 0 \end{vmatrix} = B_0 \hat{z}(+i\omega) e^{i(kx-\omega t)}$$

$$E_0 \hat{z}(ik) e^{i(kx-\omega t)} = \hat{z} B_0 (+i\omega) e^{i(kx-\omega t)}$$

$$E_0 k = B_0 \omega$$

$$E_0 = c B_0 \quad \text{since} \quad c = \frac{\omega}{k}$$

Questions:



$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

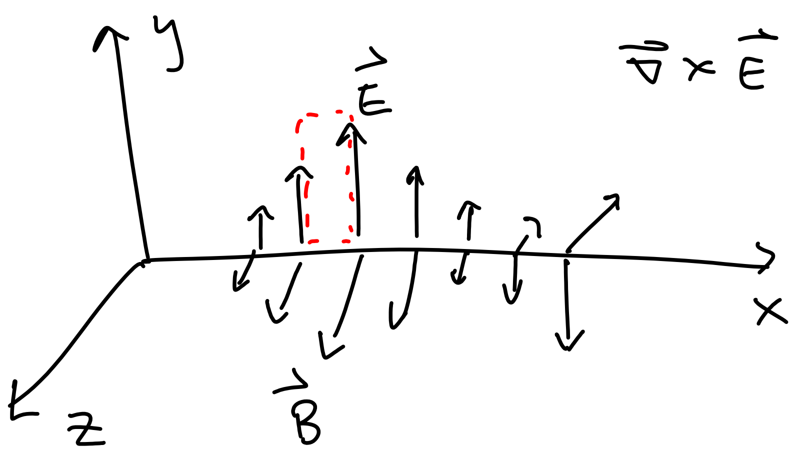
$$F_{max} = qE_0\hat{y} + q v B_0 \hat{y} = \left(qE_0 + qv\frac{E_0}{c}\right)\hat{y}$$

$$\vec{\nabla} \cdot \vec{E} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) \cdot E_0\left(0\hat{x} + \hat{y} + 0\hat{z}\right)e^{i(kx - \omega t)}$$

$$= 0$$

Hmwk: Derive results for the other differential form of ME's.

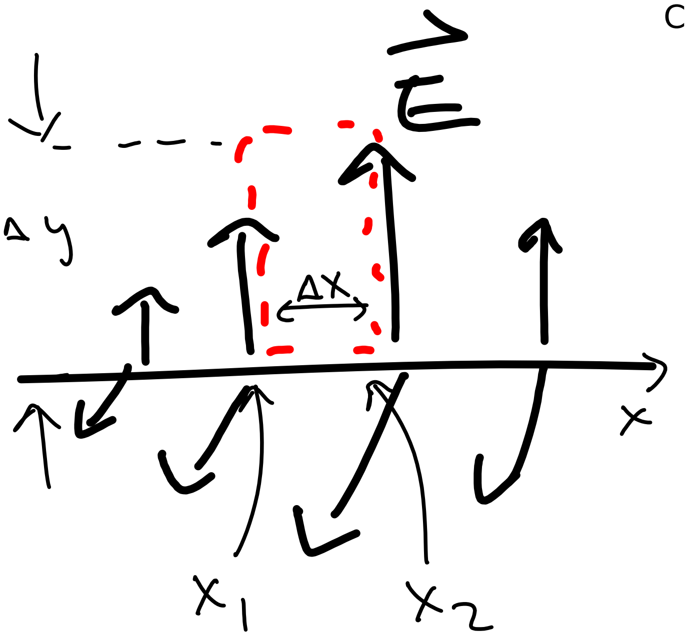
How does this soln satisfy the integral form of ME's?



$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Stokes th.

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}$$



Choose rectangular path fixed in x,y,z

Why fixed?

Partial wrt position means fix time or a snapshot.
Partial wrt time means position is fixed.

The dashed red rectange is fixed in space
while a snapshot is taken of E and B.

$$\vec{E}(x) = E_0 \hat{y} e^{i(kx - \omega t)}$$

$$\oint \vec{E} \cdot d\vec{r} = E(x_2)\Delta y - E(x_1)\Delta y$$

clockwise $\quad d\vec{r} = dy\,\hat{y} \quad \int dy \to \Delta y$

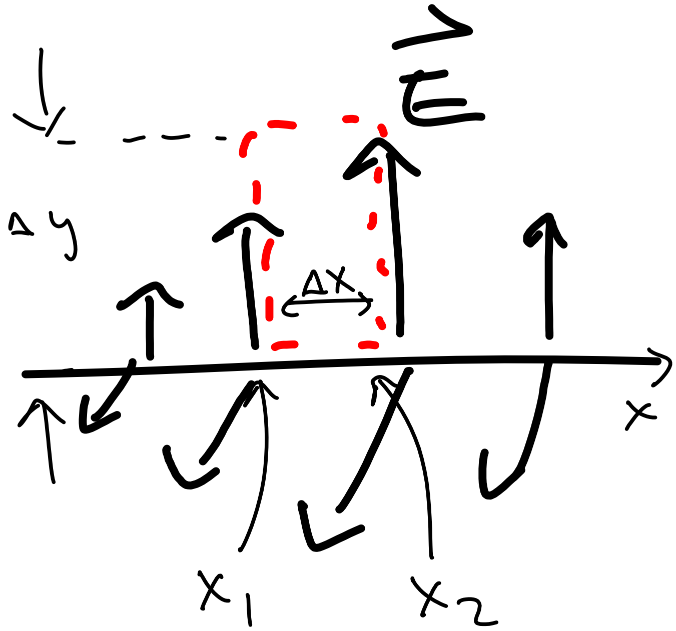
Using the right hand rule, with B out, integrate
ccw as viewed down the z-axes.

$$E(x_2) = E(x_1) + \frac{\partial E}{\partial x}\Delta x$$

fixed what?

$$\oint \vec{E}\cdot d\vec{r} = \left(E(x_1) + \frac{\partial E}{\partial x}\Delta x\right)\Delta y - E(x_1)\,\Delta y = \frac{\partial E}{\partial x}\Delta x\,\Delta y$$

$$-\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a} = \ ?$$



$$-\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a} = -\frac{\partial}{\partial t}\int B\,dx\,dy = -\frac{\partial B}{\partial t}\Delta x\,\Delta y$$

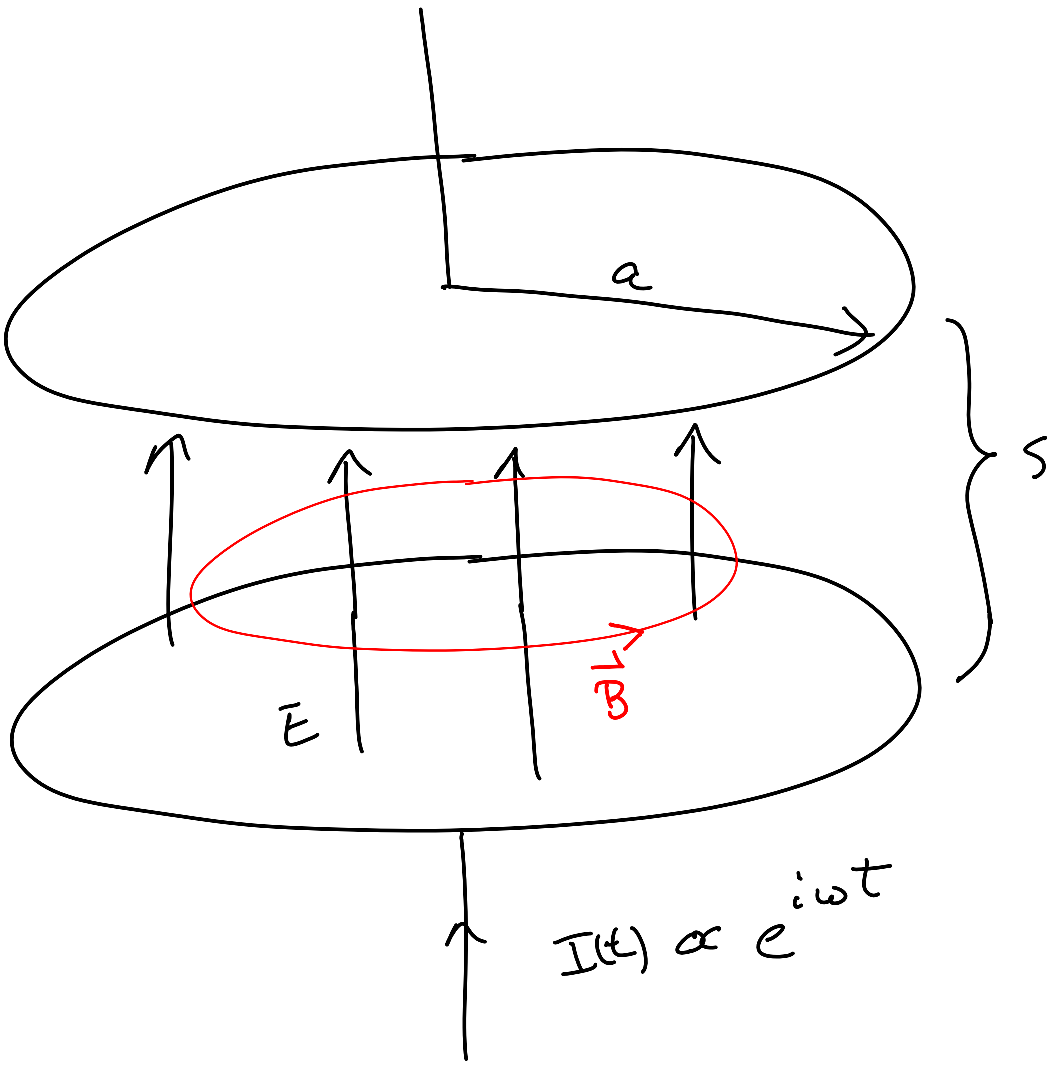
$$\frac{\partial E}{\partial x}\Delta x\,\Delta y = -\frac{\partial B}{\partial t}\Delta x\,\Delta y$$

$$\boxed{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}}$$

Hmwk: Do a similar analysis for Ampere's law deriving a similar PDE.

Combine this PDE with your to derive the wave eqn.

Example: An oscillating current drives a capacitor. We will then show (lecture and hmwk) how to obtain a perturbative soln.



$$E_{tot}(t) = E_1 e^{i\omega t} + E_2(t) + \cdots \sim e_1 + \epsilon e_2 + \epsilon^2 E_3 + \cdots$$

$\uparrow \frac{1}{c^2}$ small

$$B_{tot} = B_1 + B_2 + \cdots$$

$$E_1 e^{i\omega t} = \frac{\sigma(t)}{\epsilon_0} = \frac{1}{\epsilon_0}\frac{Q(t)}{A} \qquad \frac{\partial E}{\partial t} = E_1 (i\omega) e^{i\omega t} = \frac{1}{\epsilon_0}\frac{I}{A}$$

$$\boxed{I = \epsilon_0 A E_1 i\omega e^{i\omega t}}$$

Apply Ampere's law in vacuum between plates.

$$\oint \vec{B}\cdot d\vec{r} = \mu_0 \int \vec{J}_D \cdot d\vec{a}$$

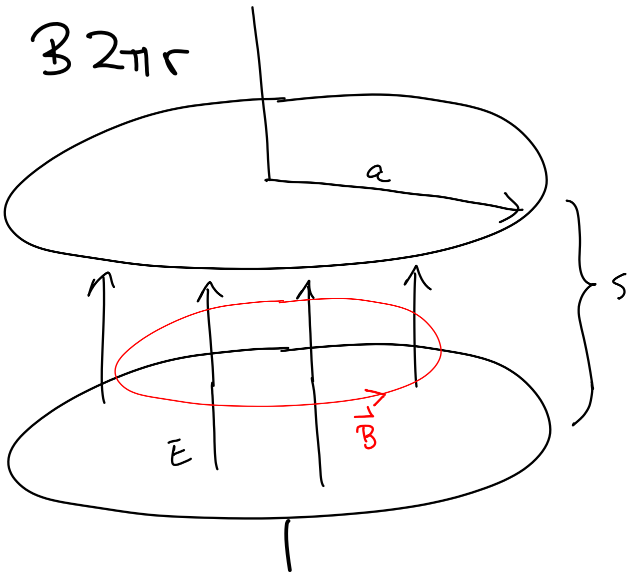
πr^2

$\epsilon_0 \frac{\partial E}{\partial t}$

$$E = \frac{\sigma}{\epsilon_0} = \frac{1}{\epsilon_0}\frac{Q}{A}$$

πa^2

$B\,2\pi r$



$$\frac{\partial E}{\partial t} = \frac{1}{\epsilon_0}\frac{1}{A} I$$

$$B\,2\pi r = \mu_0 \frac{I}{\pi a^2}\pi r^2$$

$$B = \frac{\mu_0 I}{2\pi a^2} r$$

$$B = B_1 = \frac{\mu_0 \epsilon_0}{2\pi a^2} r E_1 i\omega e^{i\omega t}$$

$$i = e^{i\pi/2}$$

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}; \quad \mu_0\epsilon_0 = \frac{1}{c^2}$$

Questions:

informational: what does the i mean?

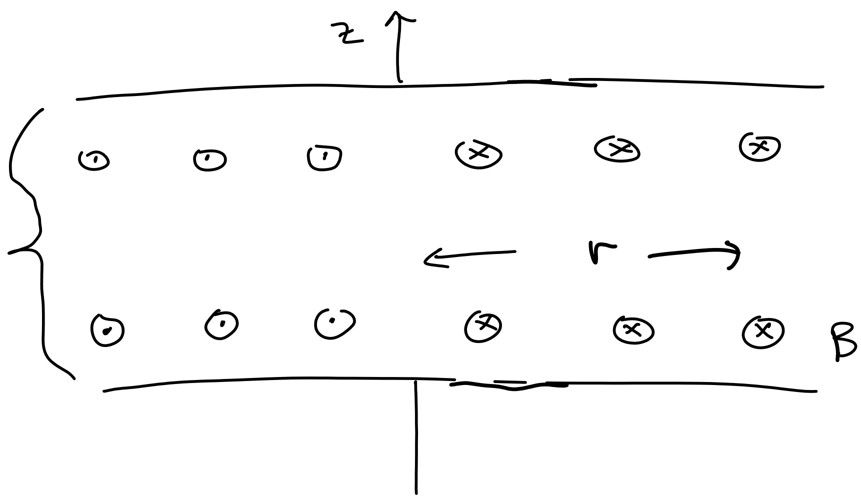
This changing B generates another E.
congruous: How do we calculate it?

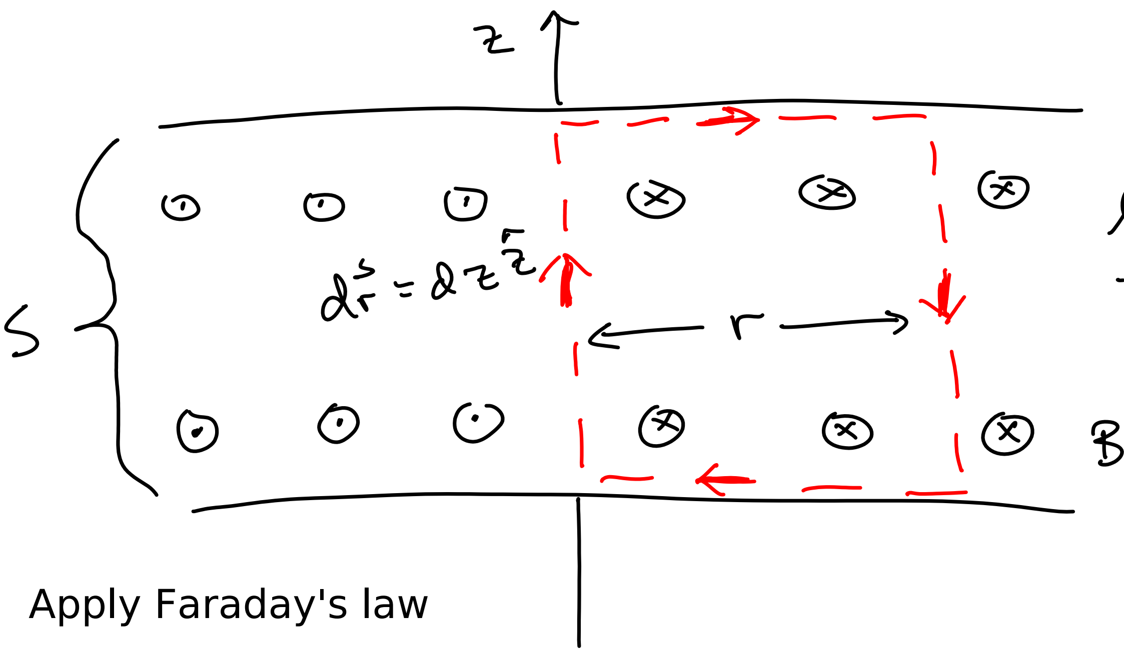
$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

stokes

$$\oint \vec{E}\cdot d\vec{r} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a}$$

What path do we use to apply Faraday's law?





let E be positive in upward ↑ direction

Apply Faraday's law

Hmwk problem on the flow of current in an Ohmic material (salt water)