$$\nabla^2 V(x,y,z) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Sep variables: assume $V(x,y,z) = X(x)\,Y(y)\,Z(z)$

find $X(x)$, $Y(y)$, $Z(z)$

$$\frac{\partial^2}{\partial x^2}(XYZ) + \frac{\partial^2}{\partial y^2}(XYZ) + \frac{\partial^2}{\partial z^2}(XYZ)$$

$$\|$$

$$\frac{d^2 Z(z)}{dz^2}$$

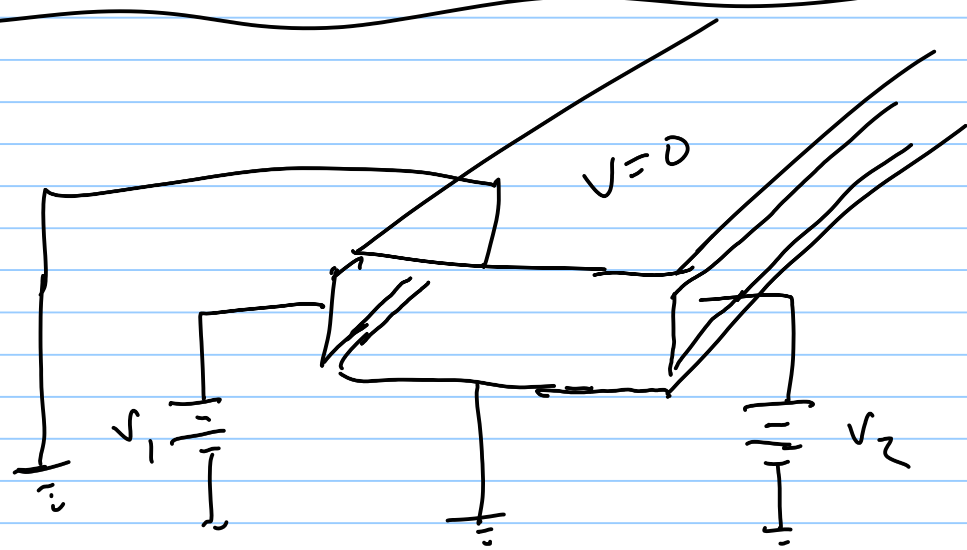
$$\frac{\cancel{Y}\cancel{Z}}{X\cancel{Y}\cancel{Z}}\frac{\partial^2 X}{\partial x^2} + \frac{\cancel{X}\cancel{Z}}{\cancel{X}Y\cancel{Z}}\frac{\partial^2 Y}{\partial y^2} + \frac{\cancel{X}\cancel{Y}}{\cancel{X}\cancel{Y}Z}\frac{\partial^2 Z(z)}{\partial z^2} = 0$$

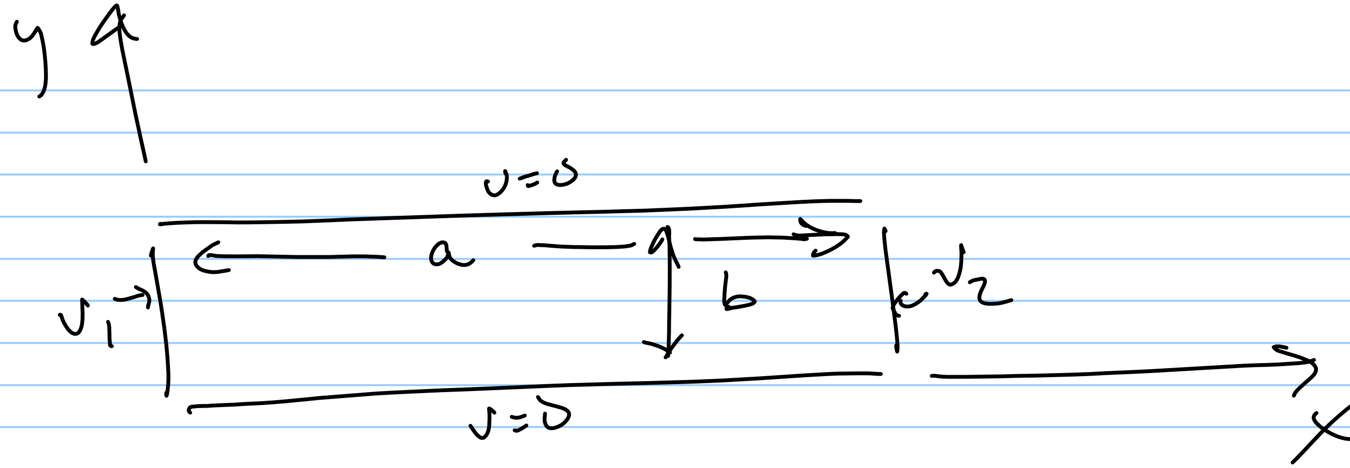
$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = 0$$

$$\underbrace{C_1}_{\text{constants}} + \underbrace{C_2}_{\text{constants}} + \underbrace{C_3}_{\text{constants}} = 0$$

1 PDE $\Longrightarrow$ 3 ODE's

Ex:





B.C. $\begin{cases} x = 0 & V = V_1 \\ x = a & V = V_2 \\ y = 0, b & V = 0 \end{cases}$

Solve 3 ODE's

$$\frac{d^2 Z(z)}{dz^2} = 0 = C_3$$

$Z(z) = \text{const}$

$$\frac{d^2 \bar{Y}(y)}{dy^2} = C_2 \bar{Y}$$

Solns
$$\begin{cases} \bar{Y} = A \sin ky + B \cos ky & \underline{C_2 < 0} \\ \bar{Y} = A' e^{\sqrt{C_2}\, y} + B' e^{-\sqrt{C_2}\, y} & \underline{C_2 > 0} \\ \bar{Y} = A'' + B'' y & \underline{C_2 = 0} \end{cases}$$

$\bar{Y}(y=0) = 0$ & $\bar{Y}(y=b) = 0$ $\quad Y = A \sin ky + B \cos ky$

$Y(0) = A \sin 0 + B \cos(0) = B \Rightarrow B = 0$

$Y(b) = 0 = A \sin kb \Rightarrow kb = n\pi \quad n = 1, 2, 3, \ldots$

$$k = \frac{n\pi}{b}$$

$$Y(y) = A \sin \frac{n\pi}{b} y \quad \Rightarrow \quad \frac{d^2 Y}{dy^2} = -k^2 Y$$

$$C_2 = -k^2$$

$$C_2 = -\left(\frac{n\pi}{b}\right)^2$$

$$C_1 + \underbrace{C_2}_{= -k^2} + \overbrace{C_3}^{0} = 0 \quad \Rightarrow \quad C_1 = k^2 > 0$$

$$\frac{d^2 X}{dx^2} = C_1 X, \quad C_1 = k^2$$

$$X(x) = A' e^{\sqrt{C_1}\, x} + B' e^{-\sqrt{C_1}\, x}$$

$$X(x) = A' e^{kx} + B' e^{-kx}$$

$$V(x,y,z) = X\,Y\,Z = \left(G e^{kx} + H e^{-kx}\right)\sin ky$$

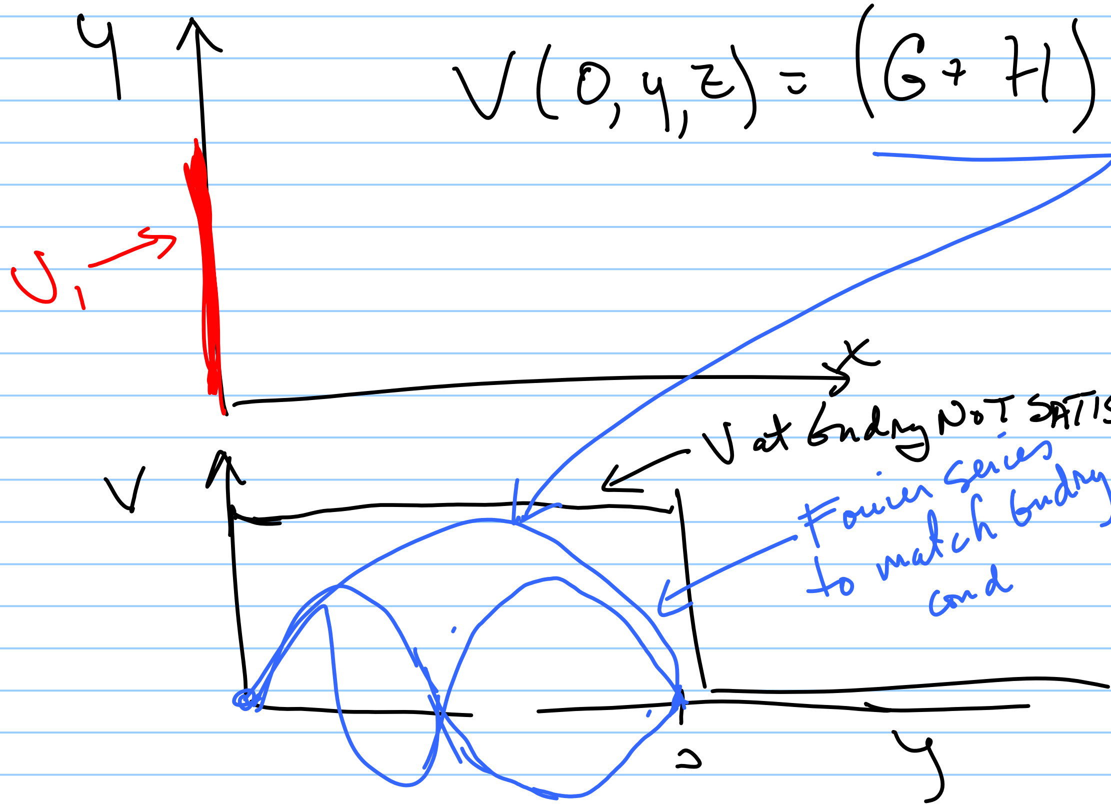
constants

This soln satisfies Laplaces eqn.

Does this soln satisfy Laplaces eqn for our boundary conditions?

Does this soln satisfy bndry at $x = 0, a$

$$V(0,y,z) = (G+H)\sin ky$$



V at bndry NOT SATISFIED!

Fourier series to match bndry cond

Superposition ⇒ to add other solns to PDE & sum is still a soln