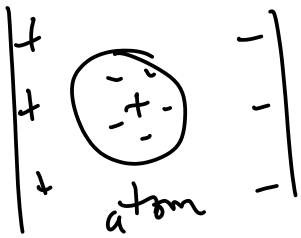
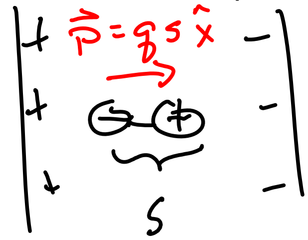
Lecture 30 Shadowitz chapter 6 Hmwk due Wed April 2

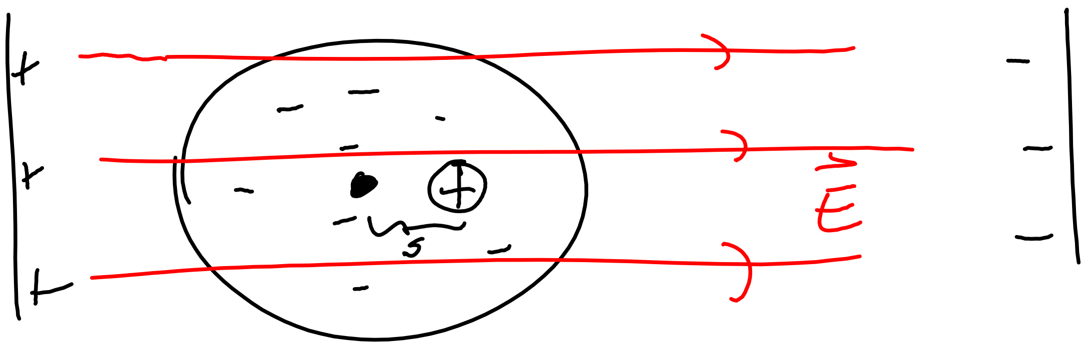


Induced dipole



Model: the minus spherical distribution is not distorted by just offset from the positive charge due to the applied electric field.

congruous: I understand the model but how do I calculate p? (don't use knee-jerk thinking).



$$p = q s$$

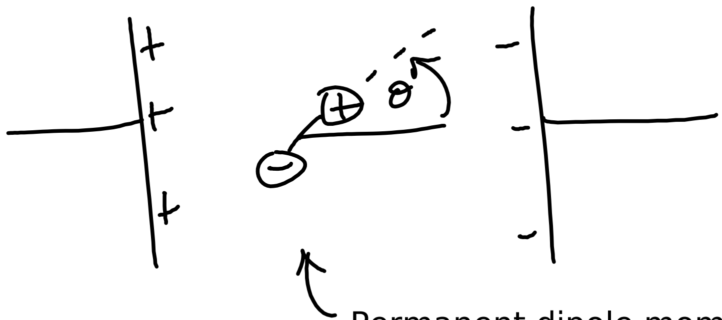
Linear approximation is $p = \alpha E_{applied}$

α is the polarizability of the atom.

Homework problem 8: Find an expression for the polarizability of the atom in terms of the atomic radius using this model. Compare this with the measured value for a hydrogen atom.
Also, compare the dipole moment of an such an atom in a typical electric field with that of the permanent dipole moment of water.

The forces between induced and permanent dipoles are fundamental to our understanding of chemistry and biology.

Example: water vapor in a capacitor at temperature T.



Causal/creative: What factors determine the percent of water molecules that are at angle theta for a given temperature?

analogous: What percent of hydrogen atoms in a gas at STP are in the first excited state?

Boltzman factor is often used to determine a frequency or number of states ratio:

$$e^{-\Delta U/kT} = \frac{\text{Number of states with energy } U_2}{\text{Number of states with energy } U_1}$$

$\Delta U = U_2 - U_1$ U_1 is hydrogen gnd state energy

U_2 is " 1st excited state energy

For the first excited state U is 13.6 eV larger than the gnd state while KT is 0.025 eV.

The number of hydrogen atoms in the first excited state divided by the number in the ground state is

$$\frac{N_1}{N_0} = e^{-13.6/.025}$$

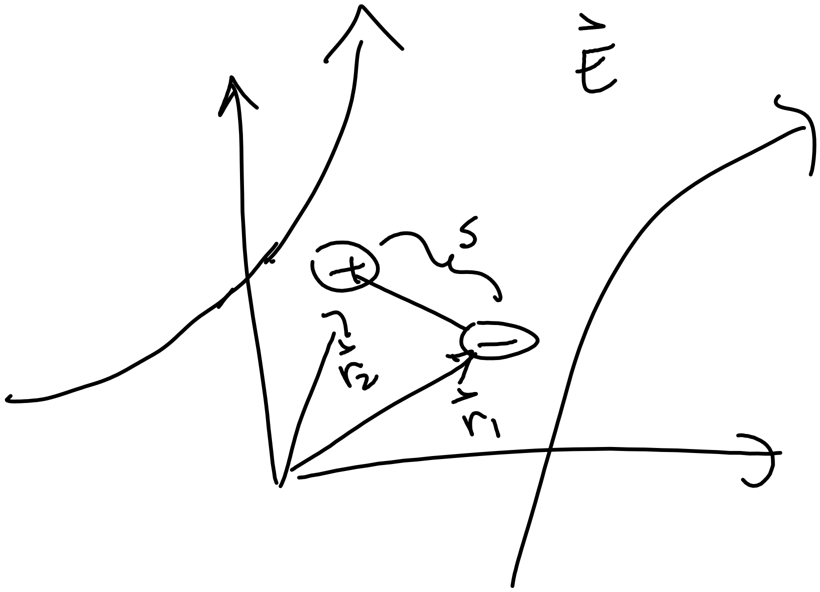
The probability that a state is occupied is $$P_1 = \frac{e^{-U_1/kT}}{Z}$$

where Z is the partition function. When taking ratios, as done above, the partition function cancels.

congruous: How do I calculate U for the dipole in an electric field?

informational: If U is small compared with KT what's P look like?

informational: If U is large compared with KT what's P look like?



$$U = qV_2 - qV_1 = q\Delta V$$

Homework problem 9: Find an expression for the potential energy U in two ways:

(1) use $\Delta V = \int_{r_1}^{r_2} \vec{\nabla} V \cdot d\vec{r}$

(2) $\vec{F} = \vec{\nabla}(\text{Potential Energy})$

and a previous homework result that

$$\vec{F} = (\vec{p} \cdot \vec{\nabla})\vec{E}$$

Sketch U vs theta to show that the minimum of the U meets your expectations.

The bound charges are determined by the dipole moment per unit volume, $\vec{P}$

That's the quantity we will always need to find in determining E in materials. It is the starting point like the position vector in mechanics.

Main points in developing a model of P for permanent dipoles.

1.) All molecules have the same permanent dipole moment in magnitude but point in different directions.

2.) Collisions randomize the dipoles making an isotropic medium.

3.) Application of E tends to orient the dipoles along the z direction (same directions as E). The dipole moment along all other directions averages to zero.

4.) We don't know for sure the exact direction of the dipole moment for all molecules but only some average value.

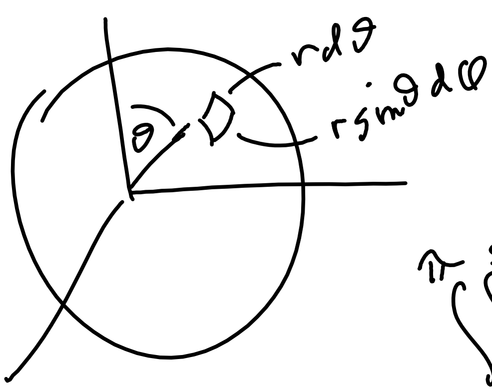
$$\vec{P}\left(\frac{\text{dipole mom}}{\text{vol}}\right) = \sum_{\substack{\text{molecules in}\\ \text{volume}}} \frac{\text{average dipole moment in z direction}}{\text{molecule}}$$

$$= \sum_{\text{theta}} \frac{\text{number of molecules along theta}}{\text{volume}} \left(\begin{array}{c}\text{dipole moment along z for a molecule}\\ \text{at angle theta}\end{array}\right)$$

$$\frac{\text{number of molecules along theta}}{\text{volume}} = \left(\frac{\text{number of molecules}}{\text{volume}}\right)\left(\begin{array}{c}\text{probability of a}\\ \text{molecule along theta}\end{array}\right)$$

$$\text{dipole moment along z for a molecule at angle theta} = p_0 \cos\theta$$

This is a 3-D problem so we use solid angles.



$$d\Omega = r^2 \sin\theta \, d\theta \, d\varphi$$

set $r = 1$

$$\text{solid angle} = \sin\theta \, d\theta \, d\varphi$$

$$\int_0^{\pi}\int_0^{2\pi} \sin\theta \, d\theta \, d\varphi = 4\pi$$

Return to the calculation

$$\vec{P}\left(\frac{\text{dipole mom}}{\text{vol}}\right) = \sum_{\text{molecules in volume}} \frac{\text{average dipole moment in z direction}}{\text{molecule}}$$

$$= \sum_{\text{theta}} \frac{\text{number of molecules along theta}}{\text{volume}} \left(\text{dipole moment along z for a molecule at angle theta}\right)$$

where $\frac{\text{number of molecules along theta}}{\text{volume}} = \left(\frac{\text{number of molecules}}{\text{volume}}\right)\left(\text{probability of a molecule along theta}\right)$ and (dipole moment along z for a molecule at angle theta) $= P_0 \cos\vartheta$

$$= \sum_{\text{solid angles}} \frac{\text{number of molecules}}{\text{volume} \cdot \text{solid angle}} \, \Delta\Omega \, P_0 \cos\vartheta$$

($\Delta\Omega$ ← solid angle)

$$\vec{P} = \int n(\vartheta)\, P_0 \cos\vartheta \, d\Omega$$

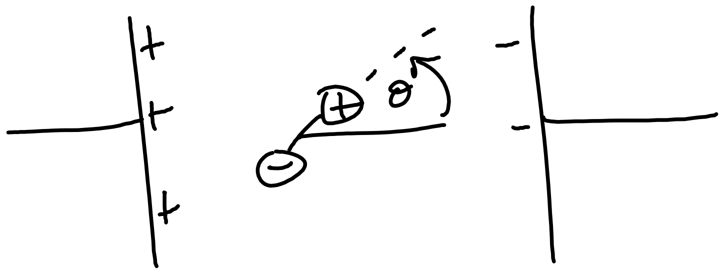
$n(\vartheta) \equiv$ number of molecules per unit solid angle per volume

Questions:

congruous: How do I calculate n which varies with theta?

Let's look at the diagram again.

If theta is pi then the molecule's dipole has the most potential energy.



Molecules can only acquire this energy due to energy exchange with the heat reservoir.

congruous: How do you calculate the probability of this occuring?

$$n(\theta) = n_0 e^{-U/kT}$$

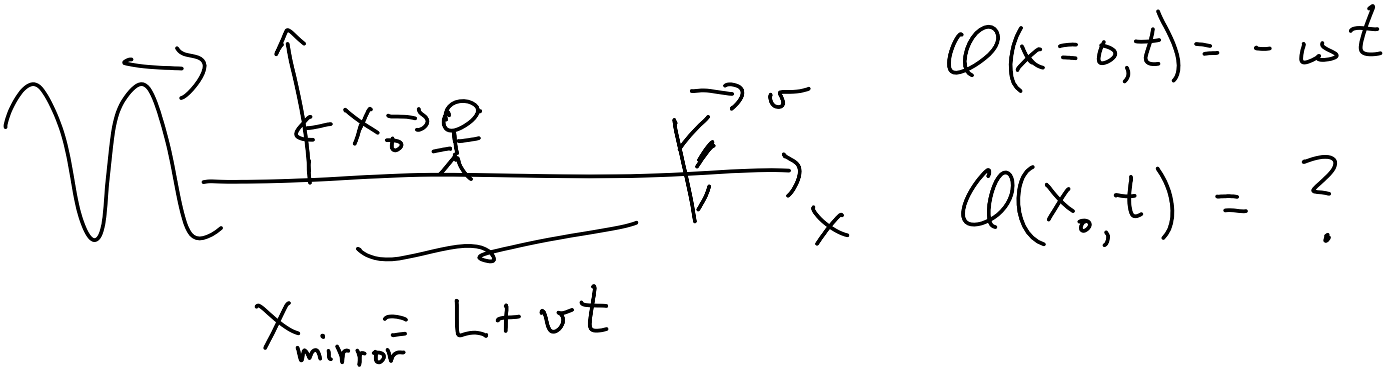
$$= n_0 e^{pE\cos\theta/kT}$$

Homework problem 10: (a) find n_0 by integrating over all solid angles in terms of N, the total number of molecules per unit volume.

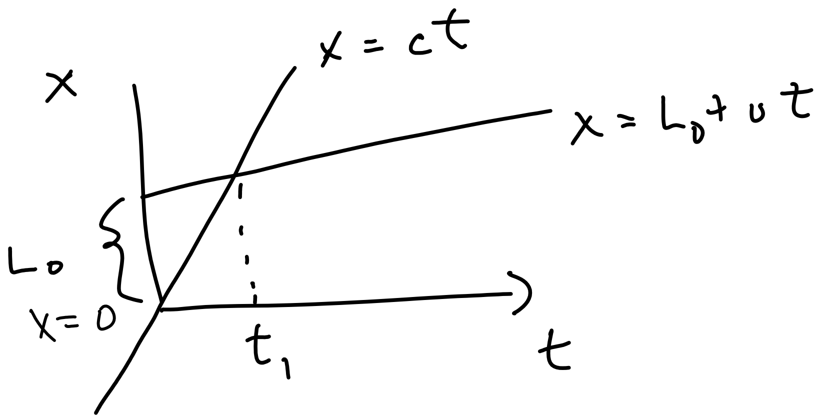
(b) assume the argument of the exponential function is small. Keep only the first two terms in this expansion. Find an expression for P in this approximation (it should depend on N, E, k, T, and the dipole moment of a molecule).

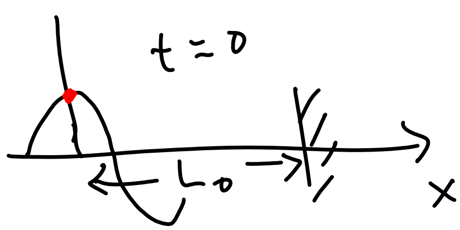
Hints on problem 6

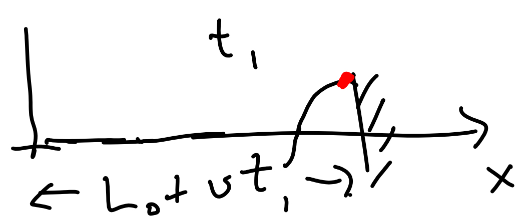
Homework problem 6.) Derive the expression for the wave reflected from a mirror moving at constant speed using the retarded time method.

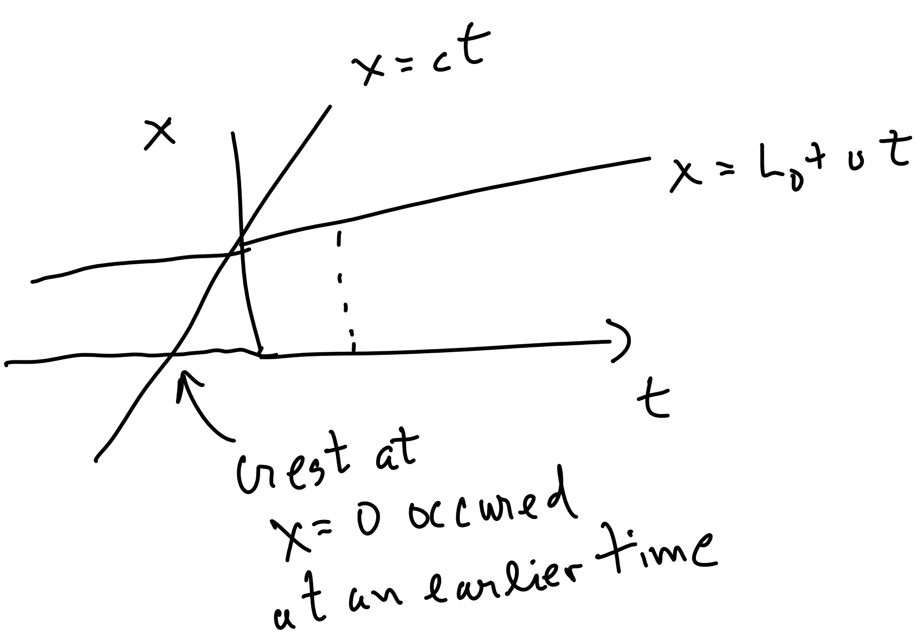


Knee-jerk thinking vs analytical thinking (writing down definitions and fundamental relations).









Let $x_{mirror}(t) = L + vt$

$$t_{retarded} = t - t_{out} - t_{return}$$

↑ now

t_{out} is the time it took the wavecrest, which reaches us now, to go from the origin to the mirror.

t_{return} is the time it took the wavecrest, which reaches us now, to go from the mirror to our location.

$$L(t - t_{return}) - x_o =$$

The LHS of the above expression is the distance the reflected wavecrest has to travel. The RHS (not shown) is another expression for this distance the crest travels (at speed c) but for what time?

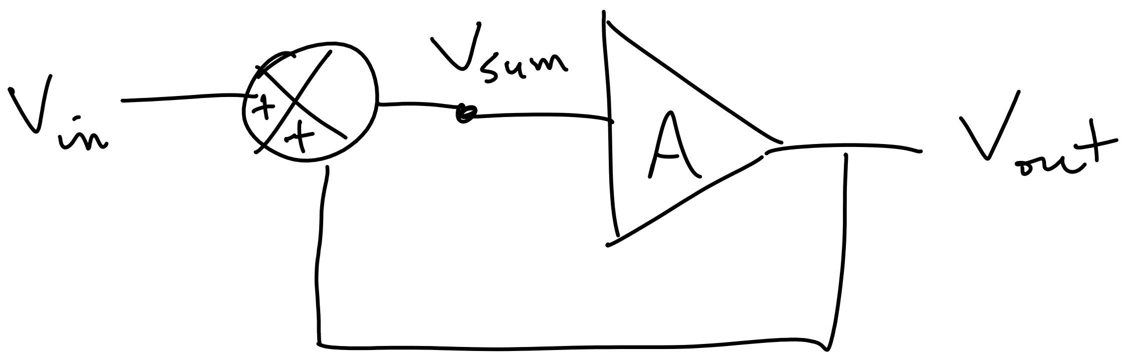
We now need an equation for the time out. The crest strikes the mirror when the mirror is located at

$$L(t - t_{return})$$

How long did it take the crest to cover this distance if it started at the origin (time out)?

↓

Circuit problem: A positive feeback amplifier's block diagram is



$$V_{sum} = V_{in} + V_{out}$$

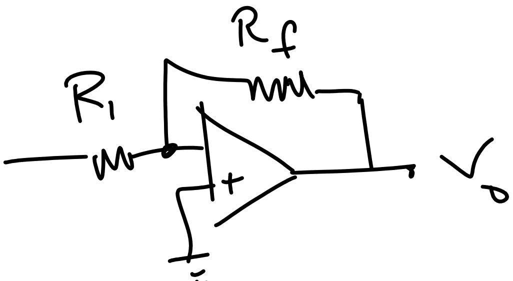
↑ pos feedback

$$V_{out} = A V_{sum} = A(V_{in} + V_{out})$$

↑ single pass gain

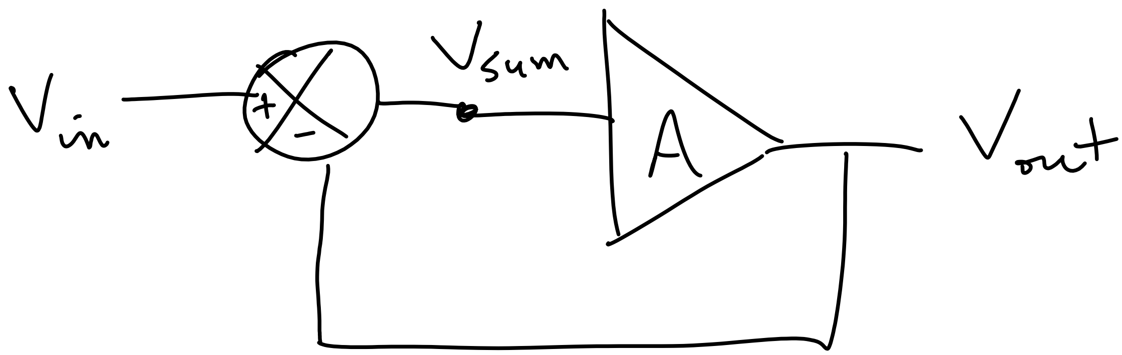
$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{A}{1-A} \simeq A(1 + A + A^2 + \cdots)$$

A is the single pass gain NOT the total gain as calculated in



$$A_{tot} = -\frac{R_f}{R_1} \qquad \frac{V_o}{V_{in}} = A_{tot}$$

Circuit problem: A negative feeback amplifier's block diagram is



$$V_{sum} = V_{in} - V_{out}$$

↑ neg feedback

$$V_{out} = A V_{sum} = A(V_{in} - V_{out})$$

↑ single pass gain

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{A}{1+A} \simeq A(1 - A + A^2 \ldots)$$