Bandwidth and pulse duration

FT pair: $f(t) = e^{-t^2/t_p^2}$ $t_p = 1/e^2$ inten. $\frac{1}{2}$ width

$F(\delta\omega) = \sqrt{\pi}\, t_p\, e^{-t_p^2 \delta\omega^2/4}$

$\Delta\omega = 2/t_p = 1/e^2$ inten. $\frac{1}{2}$ width.

$\rightarrow t_p\, \delta\omega = 2$

we usually use FWHM τ

$|f(\frac{\tau}{2})|^2 = \frac{1}{2} = e^{-2(\tau/2)^2/t_p^2}$

$\rightarrow \ln 2 = \tau^2/2t_p^2$

$\tau = \sqrt{2\ln 2}\, t_p$

likewise $\Delta\omega' = \sqrt{2\ln 2}\, \Delta\omega =$

$\tau\Delta\omega' = 2\ln 2\, t_p \Delta\omega = 4\ln 2$

$\boxed{\tau\Delta\omega = 2.78}$ FWHM

in terms of $\Delta\nu'$ (FWHM)

$\boxed{\tau\Delta\nu = 0.441}$ FWHM

mode locked HeNe: $\Delta\nu = 1\text{ GHz}$ $\tau_p = 441\text{ ps}$

Nd YAG $\Delta\sigma = 4.5\text{ cm}^{-1} \rightarrow \Delta\nu = \frac{\nu_0 \Delta\sigma}{1/\lambda_0} = c\,\Delta\sigma = 135\text{ GHz}$

$\tau = 3.25\text{ ps}$

Ti:Sapphire $\Delta\nu = 100\text{ THz} \rightarrow \tau = 4.4\text{ fs}$

Dirac delta function

$$\delta(t) = \infty \quad \text{at } t=0$$
$$= 0 \quad \text{elsewhere}$$

but
$$\int \delta(t)\,dt \equiv 1 \quad \text{unit area.}$$

δ function can be the limit of other functions

$$\lim_{t_0\to\infty} \mathcal{F}\{\mathrm{rect}(t/t_0)\} = \lim_{t_0\to\infty} t_0\,\mathrm{sinc}\left(\frac{\omega t_0}{2}\right)$$

at $\omega = 0 \to \infty$

$\omega \neq 0 \to \dfrac{\sin\left(\frac{\omega t_0}{2}\right)}{\omega/2}$

· here w/ $t_0 \to \infty$ the $\sin(\ )$ oscillates rapidly, so that any integral $\to 0$

check normalization:

$$\int_{-\infty}^{\infty} t_0\,\mathrm{sinc}\left(\frac{\omega t_0}{2}\right) d\omega = 2\int_{-\infty}^{\infty} \frac{\sin\left(\frac{\omega t_0}{2}\right)}{\omega}\,d\omega = 2\pi$$

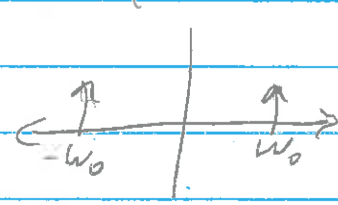
(use contour integration pole at $\omega = 0$)

$\mathcal{F}\{1\} = 2\pi\,\delta(\omega)$ $\qquad$ $\mathcal{F}\{e^{-i\omega_0 t}\} = 2\pi\,\delta(\omega-\omega_0)$

$\mathcal{F}^{-1}\{1\} = \delta(t)$ $\qquad$ $\mathcal{F}^{-1}\{e^{i\omega t_0}\} = \delta(t-t_0)$

by extension, $\mathcal{F}\{\cos(\omega_0 t)\} = \frac{1}{2}\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$

superposn of $\pm$ prop wave.



Symmetry properties of the FT

Any input function will have Re, Im parts

$$f(t) = Re(f(t)) + i\, Im(f(t))$$

each of these can be separated into odd and even parity

even $f_e(-t) \equiv f_e(t)$

odd $f_o(-t) \equiv -f_o(t)$

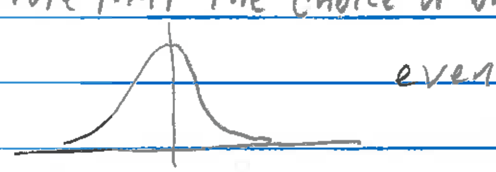
general case:

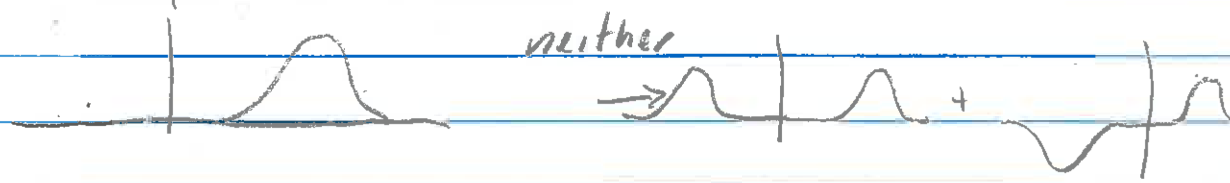
$$f(t) = f_e(t) + f_o(t)$$

$$f_e(t) = \tfrac{1}{2}(f(t) + f(-t)) \quad \text{even part}$$

$$f_o(t) = \tfrac{1}{2}(f(t) - f(-t)) \quad \text{odd part}$$

Note that the choice of origin is important:





FT

$$F(\omega) = \int \left[Re(f(t)) + i\, Im(f(t))\right]\left[\cos(\omega t) + i \sin(\omega t)\right] dt$$

we can identify $Re(F)$ and $Im(F)$

$$Re(F(\omega)) = \int \left[Re(f)\cos\omega t - Im(f)\sin(\omega t)\right] dt$$

$$Im(F(\omega)) = \int \left[Im(f)\cos\omega t + Re(f)\sin\omega t\right] dt$$

Now we can look at the parity of these components.

$\cos(\omega t)$ is even w.r.t both t, ω

$\sin(\omega t)$ is odd.

ex. $\cos \omega t$ = Real, even → Real even

$\sin \omega t$ = Real, odd → imag, odd.

Linearity

$$\mathcal{F}\{f_1(t) + f_2(t)\} = F_1(\omega) + F_2(\omega)$$

conjugate

$$\mathcal{F}\{f^*(t)\} = \int f^*(t) e^{i\omega t} dt$$

$$= \int [f(t) e^{-i\omega t}]^* dt$$

$$= F^*(-\omega)$$

Fourier transform theorems.

$$\mathcal{F}\{f(t)\} = F(\omega)$$

Scale thm:

$$\mathcal{F}\{f(at)\} = \int f(at) e^{i\omega t} dt$$

let $t' = at$ $\rightarrow \frac{1}{a} \int f(t') e^{i\frac{\omega}{a}t'} dt'$

$$= \frac{1}{a} F\left(\frac{\omega}{a}\right)$$

if $a < 0$, i.e. $t' = -|a|t$

$$\mathcal{F}\{f(at)\} = -\frac{1}{|a|} \int_{+\infty}^{-\infty} f(t') e^{i\omega\left(-\frac{t'}{|a|}\right)} dt'$$

$$= +\frac{1}{|a|} \int_{-\infty}^{+\infty} f(t') e^{i\left(\frac{\omega}{a}\right)t'} dt'$$

general case:

$$\therefore \mathcal{F}\{f(at)\} = \frac{1}{|a|} F\left(\frac{\omega}{a}\right)$$

Shift theorem:

$$\mathcal{F}\{f(t-t_0)\} = \int f(t-t_0) e^{i\omega t} dt$$

$t' = t - t_0 \rightarrow \int f(t') e^{i\omega(t'+t_0)} dt'$

$$= e^{i\omega t_0} F(\omega)$$

$$\mathcal{F}^{-1}\{F(\omega-\omega_0)\} = e^{-i\omega_0 t} f(t)$$