MATH348 - September 26, 2008 **NAME:**
Exam I - 50 Points - 50 minutes **SECTION:**

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

1. (10 Points) Conceptual Questions. Be sure to justify your responses with either words, math or both.

   (a) Suppose that the columns of $\mathbf{A}_{n\times n}$ are linearly dependent. What can you concluded about the solutions to $\mathbf{A}\mathbf{x} = \mathbf{0}$?

   (b) Suppose that you given that Rank$(\mathbf{A}_{8\times 8}) = 6$. What can you conclude about the solutions to $\mathbf{A}\mathbf{x} = \mathbf{0}$?

   (c) Suppose that $\mathbf{A}_{n\times n}$ has an eigenvalue $\lambda = 0$. What can you conclude about the solutions to $\mathbf{A}\mathbf{x} = \mathbf{0}$?

2. (10 Points) Given the following matrices and vectors:

$$\mathbf{A}_1 = \begin{bmatrix} 8 & 0 & 8 \\ 0 & 8 & 0 \\ 8 & 0 & 8 \end{bmatrix} \quad \mathbf{A}_2 = \begin{bmatrix} 0 & 8 & 0 \\ 8 & 0 & 8 \\ 0 & 8 & 0 \end{bmatrix} \quad \mathbf{b}_1 = \begin{bmatrix} 8 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} 0 \\ 8 \\ 0 \end{bmatrix} \quad \mathbf{b}_3 = \begin{bmatrix} 0 \\ 0 \\ 8 \end{bmatrix}$$

Do solutions to the following equations exist and are these solutions unique? (Yes or No)

Equation	Solutions Exist	Solutions are Unique
$\mathbf{A}_1\mathbf{x} = \mathbf{b}_1$		
$\mathbf{A}_2\mathbf{x} = \mathbf{b}_2$		
$\mathbf{A}_1\mathbf{x} = \mathbf{b}_1 + \mathbf{b}_3$		
$\mathbf{A}_2\mathbf{x} = \mathbf{b}_1 - \mathbf{b}_3$		
$(\mathbf{A}_1 + \mathbf{A}_2)\,\mathbf{x} = \mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3$		

3. (10 Points) Find $\mathbf{A}^{-1}$ given,

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 6 \end{bmatrix}.$$

4. (10 Points) Given,

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -3 & 5 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find a basis and the dimension of the null-space and column-space of $\mathbf{A}$.

5. (10 Points) Given,

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

Calculate $\mathbf{A}^4$ using the diagonalization of $\mathbf{A}$.