HW – 10 Solutions

Note Title 11/18/2007

Boas 7:12

21) FT of a Gaussian.
we did this in class. or
see pages 116-117 of my "Linear
Systems" Lecture Notes

22) There is a typo in the
Book. It says to see
ch. 12 Eq 17.4, which says

$$j_1(x) = x^1 \left(-\frac{1}{x}\frac{d}{dx}\right)^1 \left(\frac{\sin x}{x}\right)$$

$$= -\frac{d}{dx}\left(\frac{\sin x}{x}\right)$$

$$= \frac{\sin x}{x^2} - \frac{\cos x}{x} = \frac{1}{x^2}\left(\sin x - x\cos x\right)$$

in Mathematica this function
is given by

SphericalBesselJ[1, x]

13.4 claim $q(t) = CV\left(1 - e^{-t/RC}\right)$

a)

$$\dot{q} = +\frac{V}{R} e^{-t/RC}$$

so $R\dot{q} = V e^{-t/RC}$

$$\frac{q}{C} = V\left(1 - e^{-t/RC}\right)$$

$\Rightarrow R\dot{q} + \frac{q}{C} = V$ ✓

If you want to know how to solve this equation from scratch:

first solve the homog. equation

$$q_H(t) = A e^{-\alpha t} \qquad \alpha > 0$$

plug in, this gives $\alpha = 1/RC$

so $q_H(t) = A e^{-t/RC}$

This decays to zero as $t \to \infty$ so you need to add a particular solution:

$$q(t) = q_P(t) + q_H(t)$$

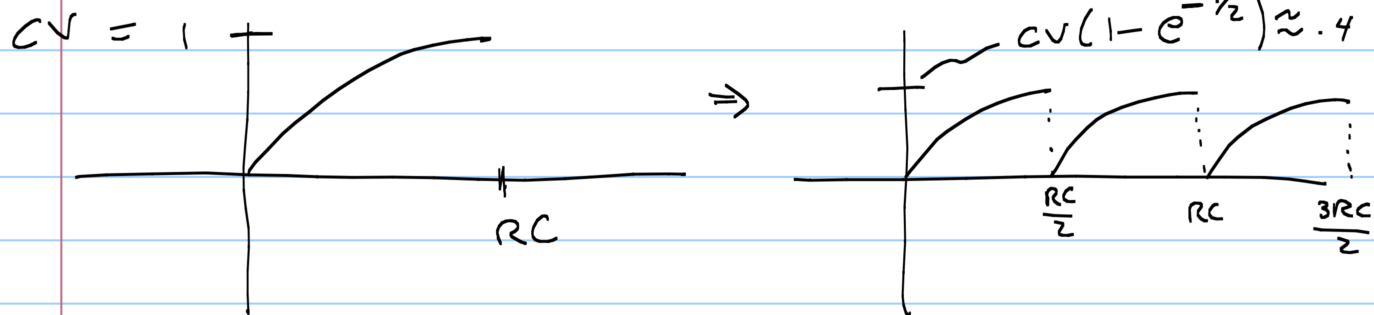
- A must have units of CV
- $q = CV$ satisfies the inhomog. inhomog. Eqn.

So $q(t) = CV + \beta CV e^{-t/RC}$

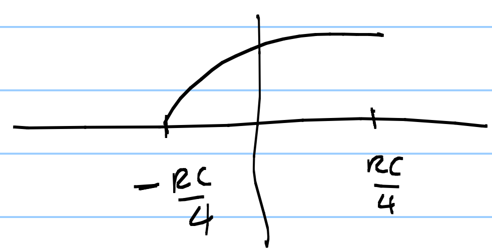
$= CV\left[1 + \beta e^{-t/RC}\right]$

where β is dimensionless

if $q(0) = 0$ then $\beta = -1$

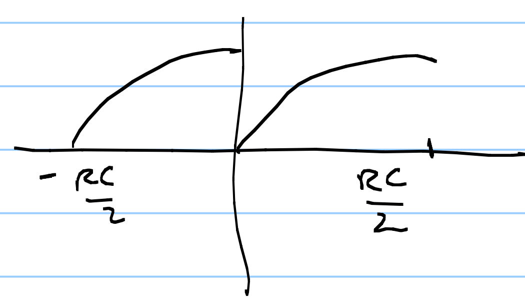


mathematica assumes symm. intervals so either shift the function in time by $RC/4$



have same periodic extension

or clone



18) $$f(x) = \int_{-\infty}^{\infty} g(\alpha)\, e^{i\alpha x}\, d\alpha$$

$$\Rightarrow f'(x) = \int_{-\infty}^{\infty} i\alpha\, g(\alpha)\, e^{i\alpha x}\, d\alpha \,.$$

So $\overline{f_1(x)}\, \frac{d}{dx} f_2(x)$

$$= \int_{-\infty}^{\infty} \overline{g_1(\alpha)}\, e^{-i\alpha x}\, d\alpha \int_{-\infty}^{\infty} i\alpha' g_2(\alpha')\, e^{i\alpha' x}\, d\alpha'$$

$$= \iint \overline{g_1(\alpha)}\; i\alpha'\; g_2(\alpha')\; e^{ix(\alpha'-\alpha)}\, d\alpha\, d\alpha'$$

integrate over x

$$\iint i\alpha'\, \overline{g_1(\alpha)}\, g_2(\alpha') \left[\underbrace{\int_{-\infty}^{\infty} e^{ix(\alpha'-\alpha)}\, dx}_{2\pi\,\delta(\alpha'-\alpha)} \right] d\alpha\, d\alpha'$$

$$2\pi \int_{-\infty}^{\infty} i\alpha\, \overline{g_1(\alpha)}\, g_2(\alpha)\, d\alpha = \int_{-\infty}^{\infty} \overline{f_1(x)}\, f_2(x)\, dx$$

if $f_1 = f_2$ (i.e $g_1 = g_2$) then

$$\int_{-\infty}^{\infty} \alpha \, |g|^2 \, d\alpha = \frac{1}{2\pi i} \int \bar{f}(x) \frac{df(x)}{dx} \, dx$$

$$= \frac{1}{2\pi i} \int \bar{f}(x) \frac{d}{dx} f(x) \, dx$$