Phys 361 Homework 9

1) (based on Pollack and Stump 7.14)

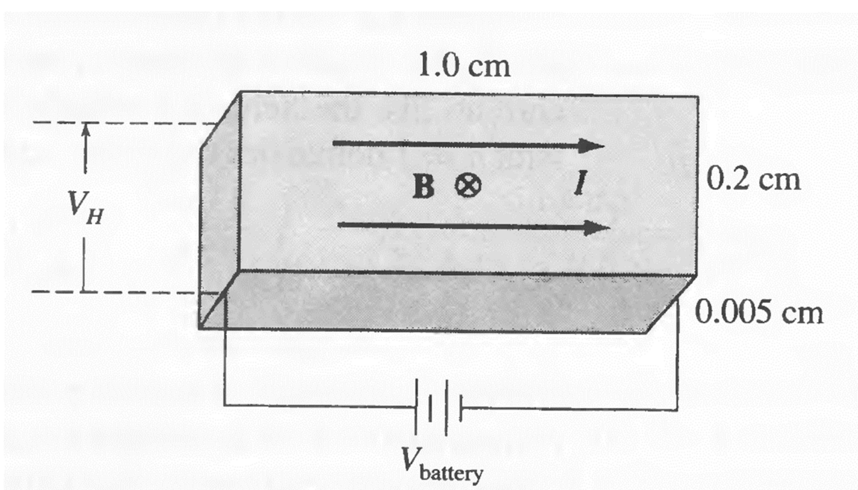
a) Find the time constant, in seconds, for the decay of charge densities in germanium and in copper. This is meant to be a simple plug & chug problem based on some material we looked at before spring break.

b) If the charges in some material take, say, 5 ns to approach rough equilibrium (so $\tau = 5$ ns), any process that takes significantly longer than 5 ns will preserve the static nature of the charge distribution, and so you'll be able to make various electrostatic approximations. Electromagnetic radiation comes in a variety of frequencies, and thus has oscillations with a variety of periods. Figure out the radiation frequencies corresponding to the relaxation times for copper and germanium, and identify which parts of the EM spectrum they come from (X-ray, radio, etc). This sort of thing can really matter if you go to do materials science work with radiation – radiation that has periods well above the relaxation time for a material can interact quite differently with the material than radiation with periods well below it.

2) (based on Pollack and Stump 8.1)

Back in Phys 200 we learned about the Hall effect. When charges move along a conductor in the presence of a magnetic field perpendicular to the current direction, they'll feel a force oriented across the material. That leads to a so-called Hall voltage – a voltage that stretches across a material perpendicular to the voltage that drives the current.

You can use this idea to make a probe for measuring magnetic fields. The Hall probe shown below has some current I flowing across a ribbon of semiconductor. The ribbon is placed in a magnetic field **B**, with **B** perpendicular to the plane of the ribbon. In a steady state situation, there's a voltage difference V_H between the top and bottom edges of the ribbon, which is proportional to the magnitude of **B**. Measuring V_H thus lets you find out an unknown **B**.



a) Let's suppose we have a probe with the dimensions shown in the picture, made of arsenic-doped silicon. The resistivity is $1.6\ \Omega \cdot \text{cm}$, and the charge carrier density is $n = 2 \times 10^{15}$ per cm^3. The applied voltage is 3 V. What will be the Hall voltage across the 0.2 cm width if the field strength is 0.1 T?

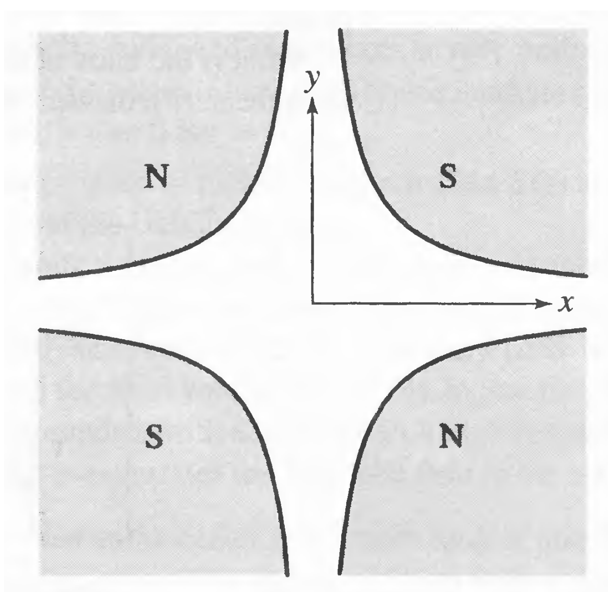
b) Show that the sign of the Hall voltage depends on the effective sign of the charge carriers – that is, the Hall voltage flips depending on whether or not the current is made up of positive charges going to the left or negative charges going to the right.

Note that we did basically this same problem in Phys 200 – it's intended to be mostly review. Don't feel the need to get too fancy, but there may be some old stuff you need to look up.

3) (based on Pollack and Stump 8.4.)

This one is fun because we get to see a quadrupole actually doing stuff, and on top of that we get to see the effect of nonuniform B-fields. Uniform B-fields are kind of boring because all they do is push charges around in little circles. Nonuniform fields are much more useful, and what we're doing (making a magnetic lens) starts to work towards the idea of magnetic confinement or a magnetic bottle.

We can use a magnetic quadrupole field as a focusing field for a charged particle beam. Take a magnet whose pole faces are as shown in the figure below:



The shaded areas are the magnet itself, and the white area in the center is empty space. There are two north poles and two south poles, and there's a charged particle beam coming out of the page. The pole faces are hyperbolas of the form $xy =$ constant. There's some interval from $z = 0$ to $z = L$ in which this quadrupole makes some nonzero field (outside of that region the field is zero). In that region, the magnetic field is given by:

$$\vec{B}(x,y,z) = b(y\hat{\imath} + x\hat{\jmath})$$

where b is a constant greater than zero. Particles enter the field region from the negative z side with some velocity $\vec{v}_0 = v_0\hat{k}$. They'll feel a force according to $q\vec{v} \times \vec{B}$.

a) Sketch the situation from the figure, and sketch the B-field lines that this quadrupole will produce.

b) Explain qualitatively why B produces focusing in the x direction and defocusing in the y direction, assuming the particles have positive charge.

c) Write the equations of motion for a beam particle of charge q and mass m, assuming the particle is subject only to magnetic forces stemming from the z component of its motion (it'll pick up velocities in the x and y direction that add additional forces, but these will be comparatively small). Solve for x as a function of z for $z > 0$, assuming $x = x_0$ and $v_x = 0$ when $z = 0$. Sketch a graph of $x(z)$.

d) From the results of part c, it should be clear that there's a value for the length L that would produce optimal focusing in x. What is that length, in terms of other given quantities?

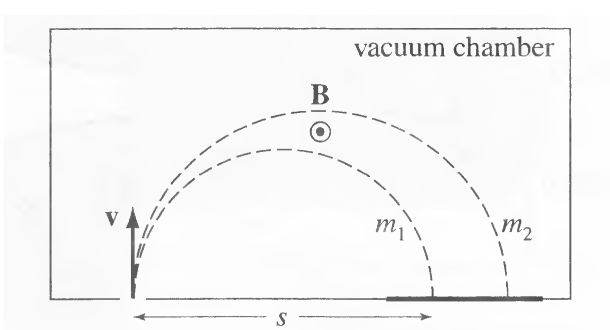
Also, just for fun, you should totally watch these two videos of allowable charged particle trajectories in a nonuniform magnetic field. They're awesome, and they're only nine seconds each:

http://www.youtube.com/watch?v=FiwqNDJuGkI

http://www.youtube.com/watch?v=Tvy7tHrbNpQ

4) (based on Pollack and Stump 8.6)

The figure below shows what's known as a sector mass spectrometer. We ionize a sample, accelerate it through some voltage, and pass the sample through a velocity selector so that all incoming ions are traveling with the same speed. Then we inject the ions into a chamber that has some magnetic field, which pushes particles around in semicircles whose radii depends on the charge-to-mass ratio of the ion. The velocity selector features an electric field of magnitude E and a magnetic field with the same B as is present in the spectrometer.



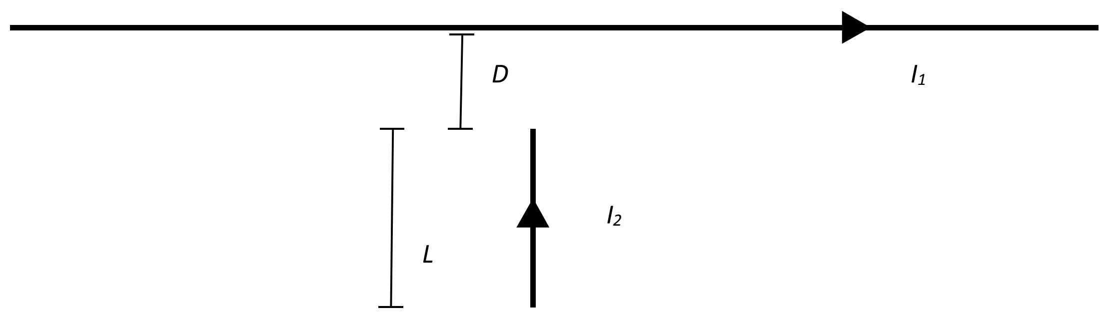
Show that the mass of some ion with charge e is given by:

$$M = \frac{eB^2 s}{2E}$$

Where s is the pictured distance – the distance between the particle input and its eventual impact on a sensor.

This kind of mass spectrometer has largely (but not entirely) been replaced by the quadrupole-based one we worked with in field session, but sector mass specs still see use in applications that require high precision. And note that we also did more-or-less this same problem in Phys 200, so don't let it get too complicated.

5) Consider two perpendicular wires as shown. There's an infinite wire with current I_1 flowing from left to right, and a wire stub of length L with current I_2 flowing towards the top of the page. There's a gap of length D between the two wires.



a) Calculate the force on wire 2 (the wire with current I_2) due to wire 1. If the everyday statement of Newton's third law holds, what should be the force on wire 1 due to wire 2?

b) Calculate the magnetic field made by wire 2 at any arbitrary point on wire 1. For clarity, let wire 2 be at $x = 0$ and find $\vec{B}(x)$ at points on wire 1 (x being the horizontal coordinate). Take the limit as $x \to 0$ and make sure $\vec{B}$ has the behavior it should have.

c) Find the magnetic force exerted on wire 1 by wire 2.

d) If you did (a) and (c) right, you'll have discovered another apparent violation of Newton's third law. In class we resolved our first such violation by noting that Newton's 3rd is really about conservation of momentum, and electromagnetic fields have momentum density that goes like $\vec{E} \times \vec{B}$, and that gave us some hope that we could patch everything up. But in this situation at first glance there's only a $\vec{B}$ and no $\vec{E}$, so that won't help. Or will it? Resolve the contradiction as best you can (qualitatively at least). Hint: You can't just punt by saying "There's no such thing physically as a wire stub." There certainly can be, under the right circumstances.