1. Prove: If the average of n numbers $x_1, x_2, \ldots, x_n$ is A, then at least one of the numbers is greater than or equal to A.

2. Are the following propositions true or false? Justify all your conclusions. If a biconditional statement is found to be false, you should clearly determine if one of the conditional statements within it is true. In that case, you should state an appropriate conclusion for this statement and prove it.

   (a) For all integers m and n, m and n are consecutive integers if and only if 4 divides $(m^2 + n^2 - 1)$.

   (b) For all integers m and n, 4 divides $(m^2 - n^2)$ if and only if m and n are both even or m and n are both odd.

3. Consider the following proposition: There are no integers a and b such that$b^2 = 4a + 2$.

   (a) Rewrite this statement in an equivalent form using a universal quantifier by completing the following:

   For all integers a and b, ...

   (b) Prove the statement in Part (a)

4. In class, we have discussed that,

   For a prime number p, if p divides xy then p divides x or p divides y.

   (a) For $x = 17$ and $y = 65$, find integers m, n such that $17m + 65n = 1$.

   (b) Let p be prime and xy be such that $p|xy$. Using conclusions from the Division Algorithm (as shown above), prove the proposition stated initially.

Using the document configuration of

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\documentclass[letterpaper,12pt]{article},
\usepackage[top=2.5cm, bottom=2.5cm, left=2cm, right=2cm]{geometry}
\usepackage{amsmath, amsfonts, amssymb, amsthm}
```

Replicate the following output:

Problem 1. The union of two sets $\mathcal{A}$ and $\mathcal{B}$ is the set of all elements that are in at least one[1] of the two sets and is designated as $\mathcal{A} \cup \mathcal{B}$. This operation is commutative $\mathcal{A} \cup \mathcal{B} = \mathcal{B} \cup \mathcal{A}$ and is associative $(\mathcal{A} \cup \mathcal{B}) \cup \mathcal{C} = \mathcal{A} \cup (\mathcal{B} \cup \mathcal{C})$. If $\mathcal{A} \subseteq \mathcal{B}$, then $\mathcal{A} \cup \mathcal{B} = \mathcal{B}$. It then follows that $\mathcal{A} \cup \mathcal{A} = \mathcal{A}, \mathcal{A} \cup \{\varnothing\} = \mathcal{A}$ and $\mathcal{U} \cup \mathcal{A} = \mathcal{U}$.

Problem 2. Applying l'Hôpital's rule, one has[2]

$$\lim_{x \to 0} \frac{\ln \sin \pi x}{\ln \sin x} = \lim_{x \to 0} \frac{\pi \frac{\cos \pi x}{\sin \pi x}}{\frac{\cos x}{\sin x}} = \lim_{x \to 0} \frac{\pi \tan x}{\tan \pi x} = \lim_{x \to 0} \frac{\pi / \cos^2 x}{\pi / \cos^2 \pi x} = \lim_{x \to 0} \frac{\cos^2 \pi x}{\cos^2 x} = 1$$

Problem 3. The gamma function Γx is defined as

$$\Gamma(x) \equiv \lim_{n \to \infty} \prod_{v=0}^{n-1} \frac{n! n^{x-1}}{x+v} = \lim_{n \to \infty} \frac{n! n^{x-1}}{x(x+1)(x+2) \cdots (x+n-1)} \equiv \int_0^\infty e^{-t} t^{x-1} \, dt$$

Problem 4. The total number of permutations of n elements taken m at a time (symbol P_n^m) is[3]

$$P_n^m = \prod_{i=0}^{m-1} (n-1) = \underbrace{n(n-1)(n-2) \ldots (n-m+1)}_{\text{total of } m \text{ factors}} = \frac{n!}{(n-m)!}$$

[1]research *cup*, *cap* and other set operators in LaTeX.
[2]research accents in LaTeX
[3]research *overbrace* and *underbrace* in LaTeX