Ex: another way is to assemble the charge a shell at a time

$dW = dq\,V$

$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$, $\quad Q = \rho\frac{4\pi r^3}{3}$

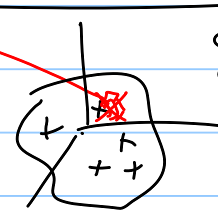
$$W = \int V\,dq = \int_0^R \frac{1}{4\pi\epsilon_0}\frac{\rho\frac{4}{3}\pi r^3}{r}\,\rho\,4\pi r^2\,dr = \frac{4}{15}\frac{\pi\rho^2}{\epsilon_0}R^5$$

Nature gives us point charges (electron is a point)

$$W = \frac{4}{15}\frac{\pi\rho^2}{\epsilon_0}R^5 \qquad \rho = \frac{q}{\frac{4}{3}\pi R^3} \qquad \rho^2 \propto \frac{q^2}{R^6}$$

$$W_{\text{ ---}} \propto \frac{q^2}{R^6}R^5 = \frac{q^2}{R}$$

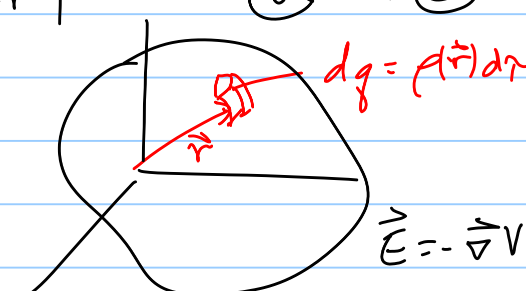
$$W = \frac{1}{2}\int V\,dq = \frac{1}{2}\int V(\underline{r})\,\rho(\underline{r})\,d\tau$$



given ρ find V

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\vec{r}\,')\,d\tau'}{|\vec{r}-\vec{r}\,'|}$$

$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$



$$W = \frac{1}{2}\int V(\vec{r})\,\rho(\vec{r})\,d\tau$$

change from ρ to $\vec{E}$

$\vec{E} = -\vec{\nabla}V$

$\vec{\nabla}\cdot\vec{E} = -\nabla^2 V = \rho/\epsilon_0$

$$W = \frac{1}{2}\int \boxed{V\,(-\epsilon_0 \nabla^2 V)}\,d\tau$$

scalar function

identity in front of book $\quad \vec{\nabla}\cdot(f\vec{A}) = f\,\vec{\nabla}\cdot\vec{A} + \vec{A}\cdot\vec{\nabla}f$

let $f = V \qquad \vec{A} = \vec{\nabla}V$

$$\vec{\nabla}\cdot(V\vec{\nabla}V) = V\,\underbrace{\vec{\nabla}\cdot(\vec{\nabla}V)}_{\nabla^2 V} + \vec{\nabla}V\cdot\vec{\nabla}V$$

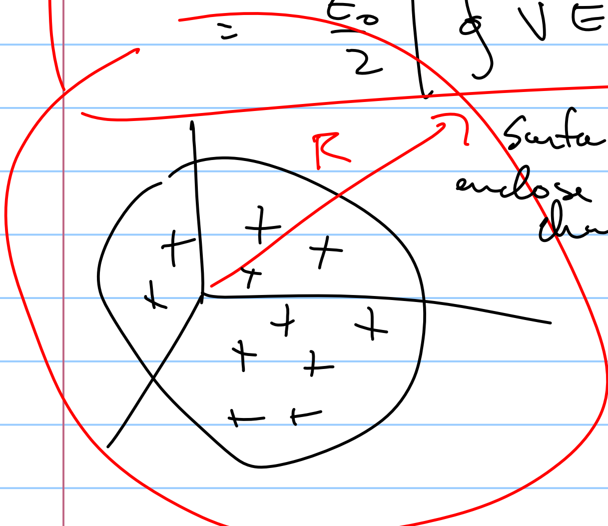
$$\boxed{V\nabla^2 V} = \vec{\nabla}\cdot(V\vec{\nabla}V) - (\vec{\nabla}V)^2$$

$$W = -\frac{\epsilon_0}{2}\left[\int \vec{\nabla}\cdot\overbrace{(V\underbrace{\vec{\nabla}V}_{-E})}^{\vec{B}}\,d\tau - \int(\underbrace{\vec{\nabla}V}_{-E})^2\,d\tau\right]$$

$$\int \vec{\nabla}\cdot\vec{B}\,d\tau = \oint \vec{B}\cdot da \quad \text{divergence thm}$$

$$W = -\frac{\epsilon_0}{2}\left[\oint (V\vec{\nabla}V)\cdot d\vec{a} - \int(-\vec{E})^2\,d\tau\right]$$

$$\boxed{= \frac{\epsilon_0}{2}\left[\oint V\vec{E}\cdot d\vec{a} + \int E^2\,d\tau\right]}$$



Surface has to enclose all charge

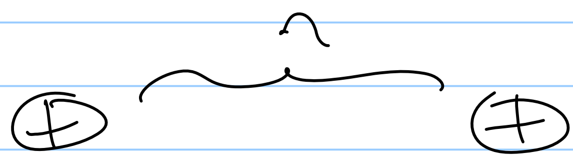
$$\boxed{\frac{1}{2}\int V\rho\,d\tau}$$

work is due charge & integrate where the charge is located as $R \to \infty$ $\quad V \propto \frac{1}{R} \quad E \propto \frac{1}{R^2}$

$da \propto R^2 \, d\Omega \qquad V E \, da \propto \frac{1}{R} \to 0$ as $R \to \infty$

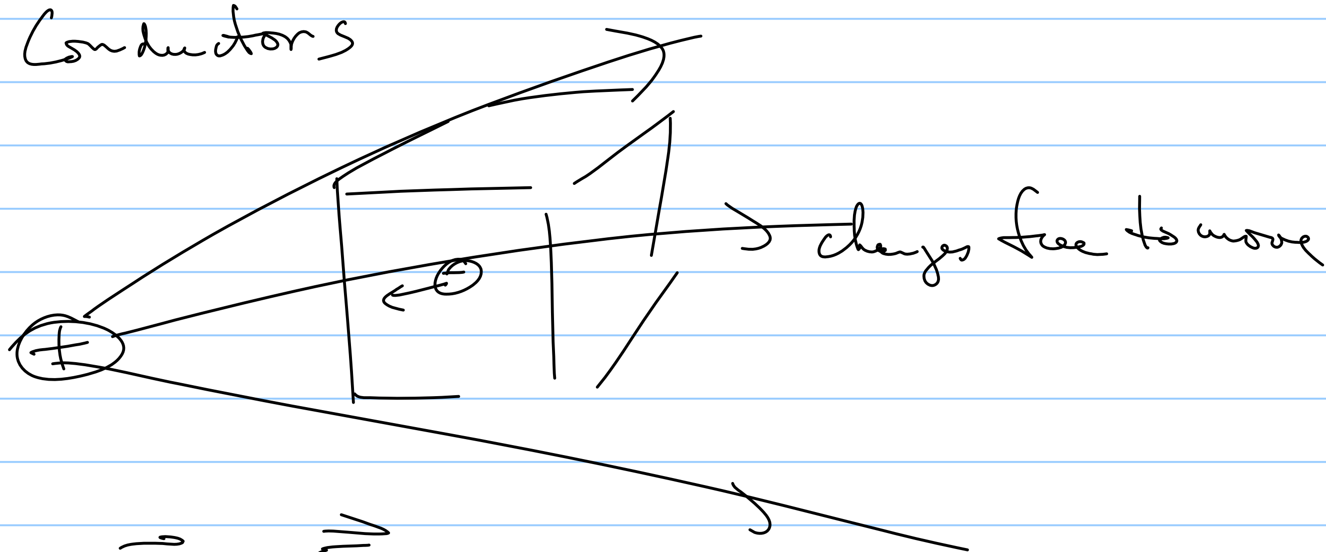
$\Rightarrow \oint V \vec{E} \cdot d\vec{a} \to 0$ as $R \to \infty$

$$\boxed{W = \frac{1}{2} \epsilon_0 \int E^2 \, d\tau}$$



$$W = \overset{\text{self energy}}{\infty} + \frac{q^2}{4\pi\epsilon_0 r}$$

Conductors



$\vec{F} = q\vec{E}$