MATH 348 - Advanced Engineering Mathematics Final Exam - Review

November 30, 2009 Final Exam: December 14, 2008

The final exam will be held on December 14th in BB201 and 204 (see ticc.mines.edu for details) and is to be taken with no notecards or calculators. There will be ten questions five of which will be from previous material. Specifically, there will be a question asking you to find the general solution to a linear system of equations and interpret this solution geometrically.¹ Also, from the linear algebra section there will be a question where you will need to find the eigenvalues and eigenvectors of a square matrix. ² Lastly, from the Fourier series section there will be questions where you will need to find a real Fourier series representation (FSR), complex FSR and a Fourier transform given some function of x. The remaining questions will be associated with partial differential equations.

- The Homogenous Heat Equation : $u_t = c^2 \nabla^2 u$.
- The Homogenous wave Equation : $u_{tt} = c^2 \nabla^2 u$.
- Laplace's Equation : $\nabla^2 u = 0$.

Furthermore, each student should be familiar with the boundary conditions and initial conditions necessary for finding unique separable solutions.³ This will naturally require a firm understanding of:

- Separation of variables
- Solutions to Boundary Value Problems
- Fourier sine and cosine series
- Solutions to constant-linear in-homogeneous ODE
- Power Series Solution to ODE

The following concepts/techniques will not appear on the exam:

- Conservation Laws
- Vibrations on a Thin Circular Membrane
- Bessel Functions
- Gamma Functions

The following is a list of concepts and methods which you should be familiar with.

12.1 Partial Differential Equations - Terminology

From this section the student should understand:

- The terms, linear, homogenous and order associated with a PDE.
- The concept superposition of solutions to a PDE.

From this section the student should be able to:

- Check to see if a given function is a solution to a PDE.
- Determine the type, order, homogeneity, and linearity of the PDE.
- Apply the rule of superposition.

¹You may be required to understand this in terms of the inverse matrix theorem and/or column/null space concepts. It is implied that you know the four representations of linear systems.

²This may be in the context of diagonalization or it may not.

³It is also important to know what these boundary conditions imply about the physical problem.

12.3 Wave equation and its solution via Fourier Series.

From this section the student should understand:

- The physical problem.
- How the physical problem is modeled by the PDE and its boundary and initial conditions.
- The solution to the wave equation on a bounded domain.

From this section the student should be able to:

- Understand the physical interpretation of the mathematical model.
- Solve the wave equation defined on bounded physical domain via Fourier Series.
- Discuss the physical interpretations of the solution to the wave equation.

12.5 Heat equation and its solution via Fourier Series.

From this section the student should understand:

- The physical problem.
- How the physical problem is modeled by the PDE and its boundary and initial conditions.
- The solution to the heat equation on a bounded domain.

From this section the student should be able to:

- Understand the physical interpretation of the mathematical model.
- Solve the heat equation defined on bounded physical domain via Fourier Series.
- Discuss the physical interpretations of the solution to the heat equation.