Interference and interferometers

Interference

Fizeau wedge

Newton’s rings

Paraxial approximation to spherical wave

Shearing interferometer

Interference in a tilted window

Fabry-perot interferometer

Multilayer mirror and filter design

Interference: ray and wave pictures

- Interference results from the sum of two waves with different *phase*:

$$E_{tot}\left(\Delta\phi\right) = E_1 e^{ikz} + E_2 e^{ikz+\Delta\phi}$$

- We measure intensity, which leads to interference

$$I_{tot}\left(\Delta\phi\right) \propto \left|E_1 e^{ikz} + E_2 e^{ikz+\Delta\phi}\right|^2 = \left|E_1 + E_2 e^{i\Delta\phi}\right|^2$$

$$= I_1 + I_2 + \sqrt{I_1 I_2}\, e^{i\Delta\phi} + \sqrt{I_1 I_2}\, e^{-i\Delta\phi}$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\Delta\phi\right)$$

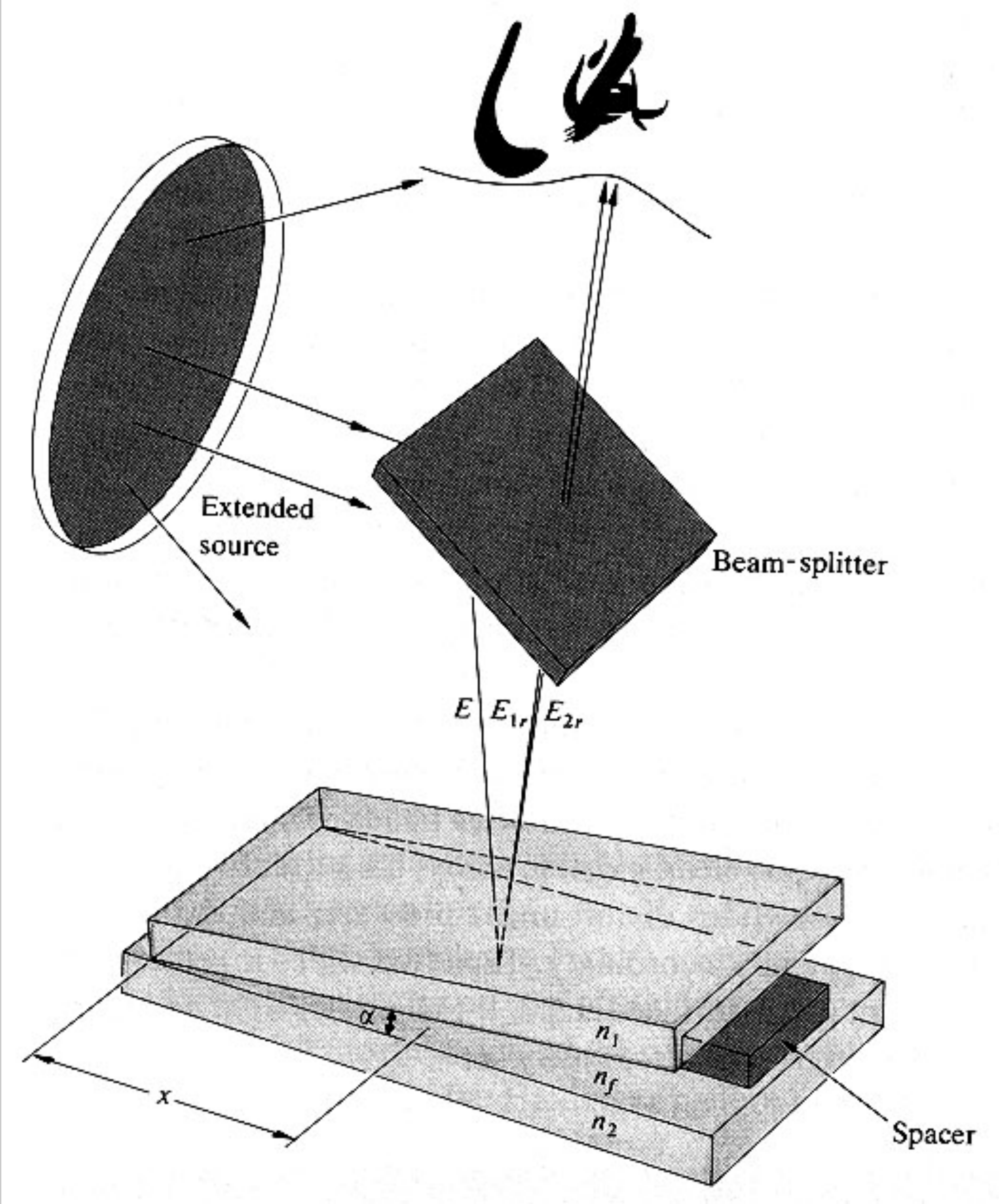
  - For the case where $I_1 = I_2$,

$$I_{tot}\left(\Delta\phi\right) = 2I\left(1 + \cos\left(\Delta\phi\right)\right) = 4I\cos^2\left(\Delta\phi / 2\right)$$

- How to generate, calculate phase difference?

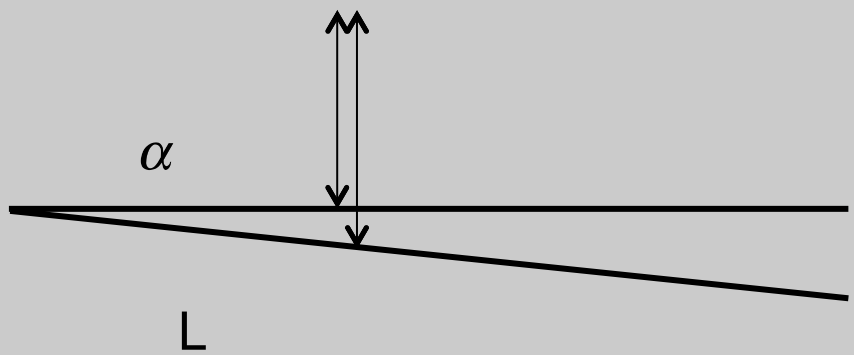
The Fizeau Wedge Interferometer

The Fizeau wedge yields a complex pattern of variable-width fringes, but it can be used to measure the wavelength of a laser beam.



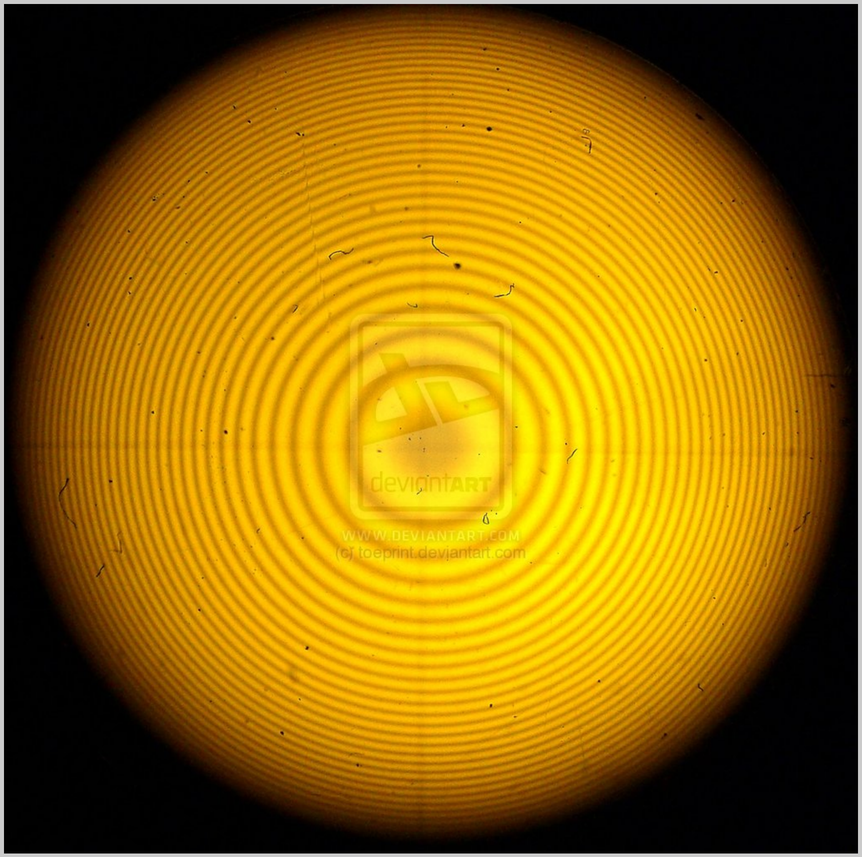
Fizeau wedge calculation

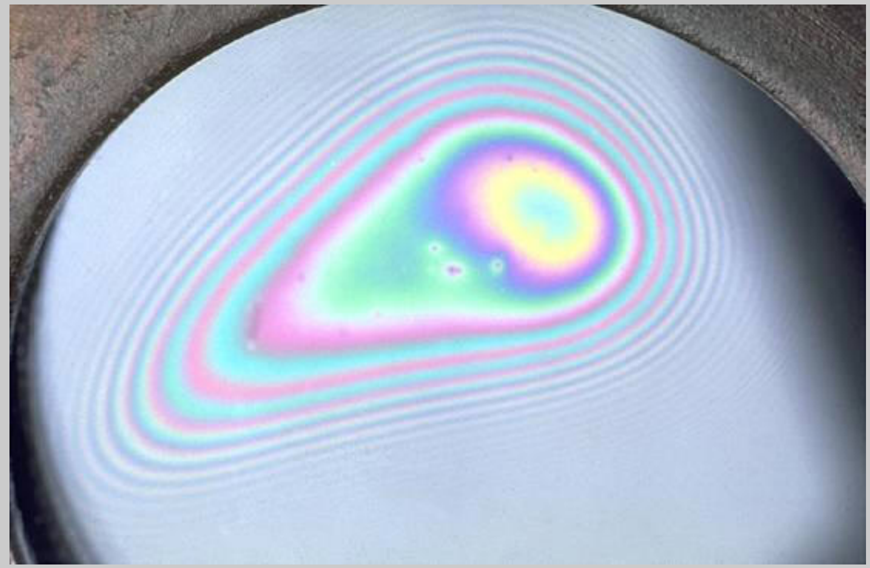
- Interference between reflections from internal surfaces



  - Angle is very small, neglect change in direction
  - Path difference: $\Delta l = 2L\sin\alpha \approx 2L\alpha$
  - Phase difference: $\Delta\phi = \dfrac{\omega}{c} n\,\Delta l \approx 2\pi \dfrac{2L}{\lambda}\alpha$ n = 1
  - Interference: $I_{tot}\left(\Delta\phi\right) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\Delta\phi\right)$
  - One fringe from one max to the next, so maxima are at $\Delta\phi = 2\pi m$
- In this interferometer, minimum path = 0, we can measure absolute wavelength: $\Delta\phi = 2\pi m = 2\pi \dfrac{2L}{\lambda}\alpha \rightarrow \lambda = \dfrac{2L}{m}\alpha$

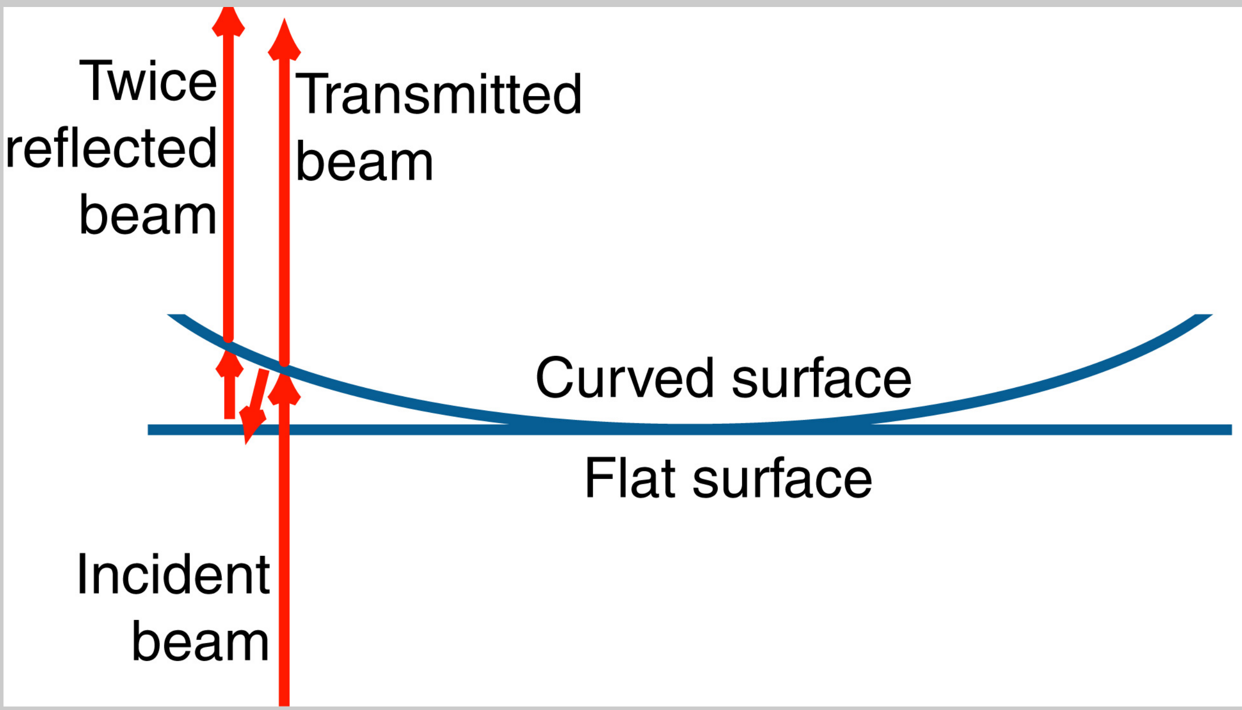
Newton's Rings





Newton's Rings

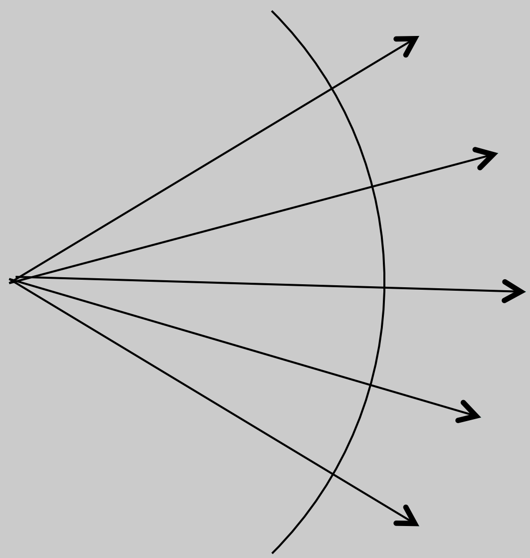
Get constructive interference when an integral number of half wavelengths occur between the two surfaces (that is, when an integral number of full wavelengths occur between the path of the transmitted beam and the twice reflected beam).



This effect also causes the colors in bubbles and oil films on puddles.

Curved wavefronts

- Rays are directed normal to surfaces of constant phase
  - These surfaces are the wavefronts
  - Radius of curvature is approximately at the focal point



- Spherical waves are solutions to the wave equation (away from r = 0)

$$\nabla^2 E + \frac{n^2\omega^2}{c^2}E = 0$$

$$E \propto \frac{1}{r}e^{i(\pm kr - \omega t)}$$

Scalar r
+ outward
- inward

$$I \propto \frac{1}{r^2}$$

Paraxial approximations

- For **rays**, paraxial = small angle to optical axis
  - Ray slope: $\tan\theta \approx \theta$
- For **spherical waves** where power is directed forward:

$$e^{ikr} = \exp\left[ik\sqrt{x^2+y^2+z^2}\right]$$

$$k\sqrt{x^2+y^2+z^2} = kz\sqrt{1+\frac{x^2+y^2}{z^2}} \approx kz\left(1+\frac{x^2+y^2}{2z^2}\right)$$

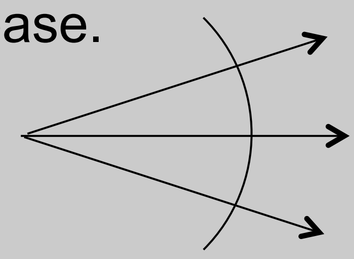
Expanding to 1st order

$$e^{i(kr-\omega t)} \rightarrow e^{ikz}\exp\left[i\left(k\frac{x^2+y^2}{2z}-\omega t\right)\right]$$

z is radius of curvature = *R*

Wavefront = surface of constant phase
For x, y >0, t must increase.
Wave is diverging:

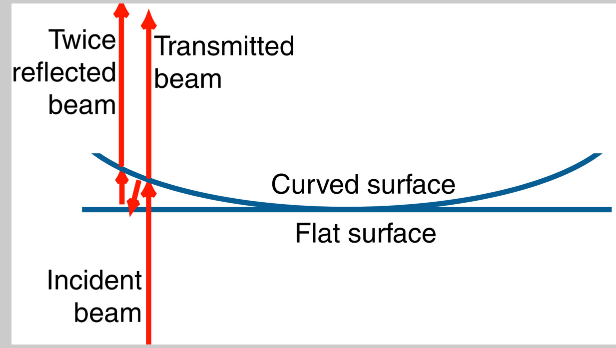
$$\phi = 0 \rightarrow k\frac{x^2+y^2}{2R} = \omega t$$



Newton's rings: interfere plane and spherical waves

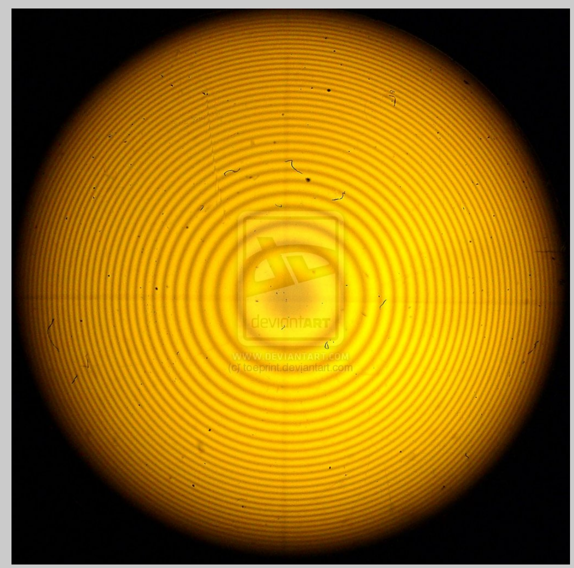
- Add two fields:

$$E(r) = E_0 + E_0 e^{i\frac{kr^2}{2R}}$$



  - Assume equal amplitude
  - For Newton's rings, 2x phase shift
- Calculate intensity:

$$I(r) \propto \left| E_0 + E_0 e^{i\frac{kr^2}{2R}} \right|^2 = 2E_0^2 + 2E_0^2 \cos\left(\frac{kr^2}{2R}\right)$$

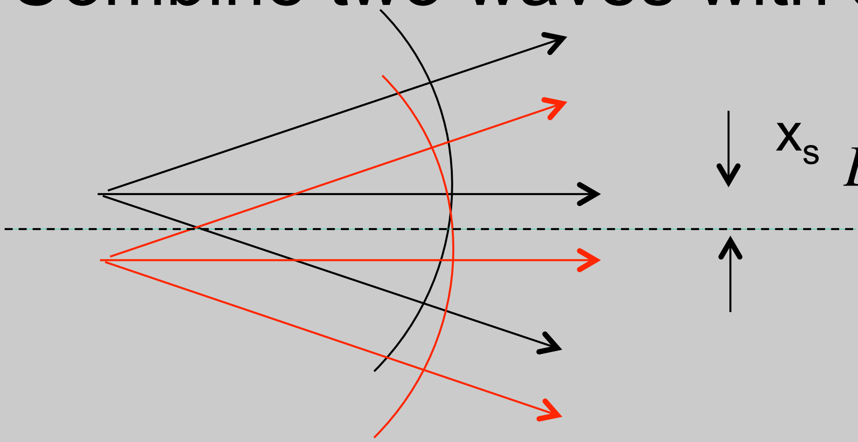


  - Local k increases with r

$$\cos\left(\frac{kr}{2R} r\right) = \cos\left(k_{local} r\right)$$

Shearing interferometer

- Combine two waves with a lateral offset ("shear")



$$E_{tot}(x) = E_0 \exp\left[i\frac{k}{2R}\left((x-x_s)^2 + y^2\right)\right] + E_0 \exp\left[i\frac{k}{2R}\left((x+x_s)^2 + y^2\right)\right]$$

$$I_{tot}(x) = I_0\left(2 + \exp\left[i\left(\frac{k(x-x_s)^2}{2R} - \frac{k(x+x_s)^2}{2R}\right)\right] + c.c.\right)$$

$$(x-x_s)^2 - (x+x_s)^2 = x^2 - 2xx_s + x_s^2 - \left(x^2 + 2xx_s + x_s^2\right) = 4xx_s$$

$$I_{tot}(x) = 2I_0\left(1 + \cos\left[\frac{2kx_s}{R}x\right]\right)$$

Fringes are straight, equally spaced
Combine with constant tilt in y direction:
leads to fringe *rotation* with divergence

Tilted window: ray propagation

- Calculate phase shift caused by the insertion of the window into an interferometer.
- Ray optics:
  - Add up optical path for each segment
  - Subtract optical path w/o window

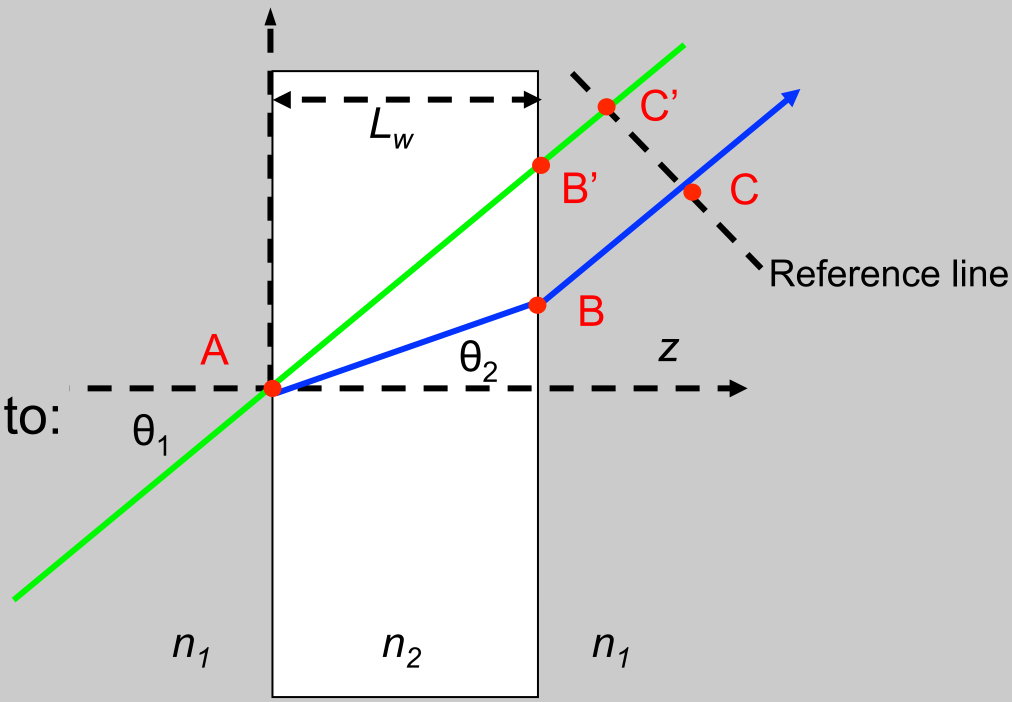
$$\Delta d = nL_{AB} + L_{BC} - L_{AB'} - L_{B'C'}$$

$$L_{AB} = \frac{L_w}{\cos\theta_2} \qquad L_{AB'} = \frac{L_w}{\cos\theta_1}$$

$$L_{BC} = L_{B'C'} + L_{BB'}\sin\theta_1$$

  - Use Snell's Law to reduce to:

$$\Delta d = n\,L_w\cos\theta_2 - L_w\cos\theta_1$$



Tilted window: wave propagation

- Write expression for tilted plane wave

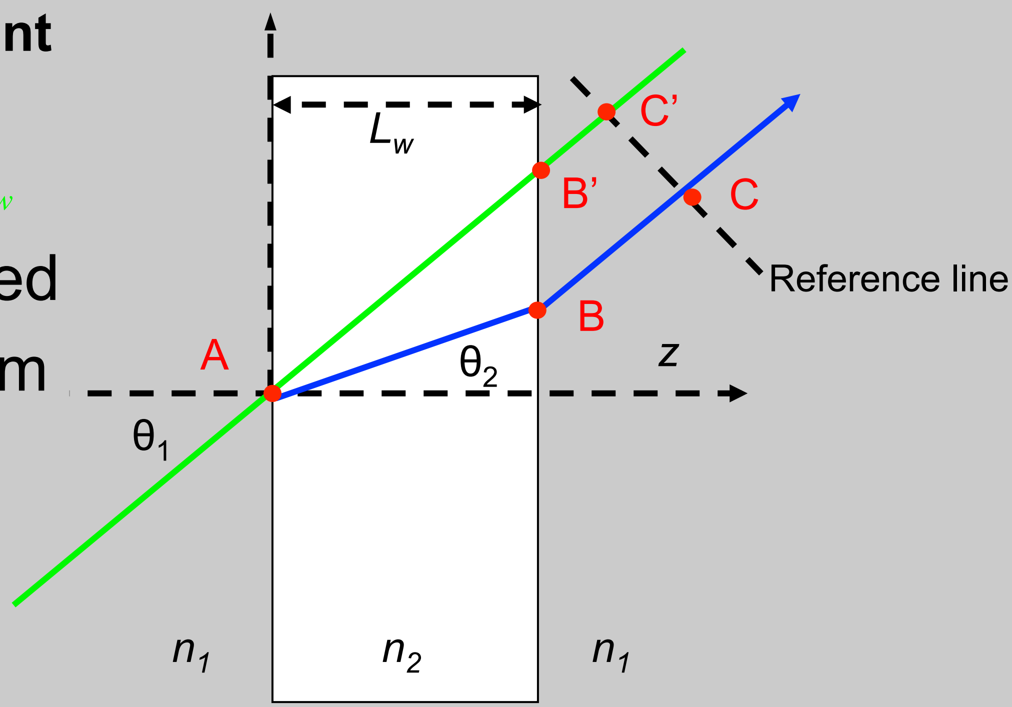
$$E(x,z) = E_0 \exp\left[i\left(k_x x + k_z z\right)\right] = E_0 \exp\left[i\frac{\omega}{c}n\left(x\sin\theta_2 + z\cos\theta_2\right)\right]$$

- Snell's Law: phase across surfaces is conserved

$$k_x x = \frac{\omega}{c} n \sin\theta$$ **is constant**

$$\Delta\phi = \left(k_2\cos\theta_2\right)L_w - \left(k_1\cos\theta_1\right)L_w$$

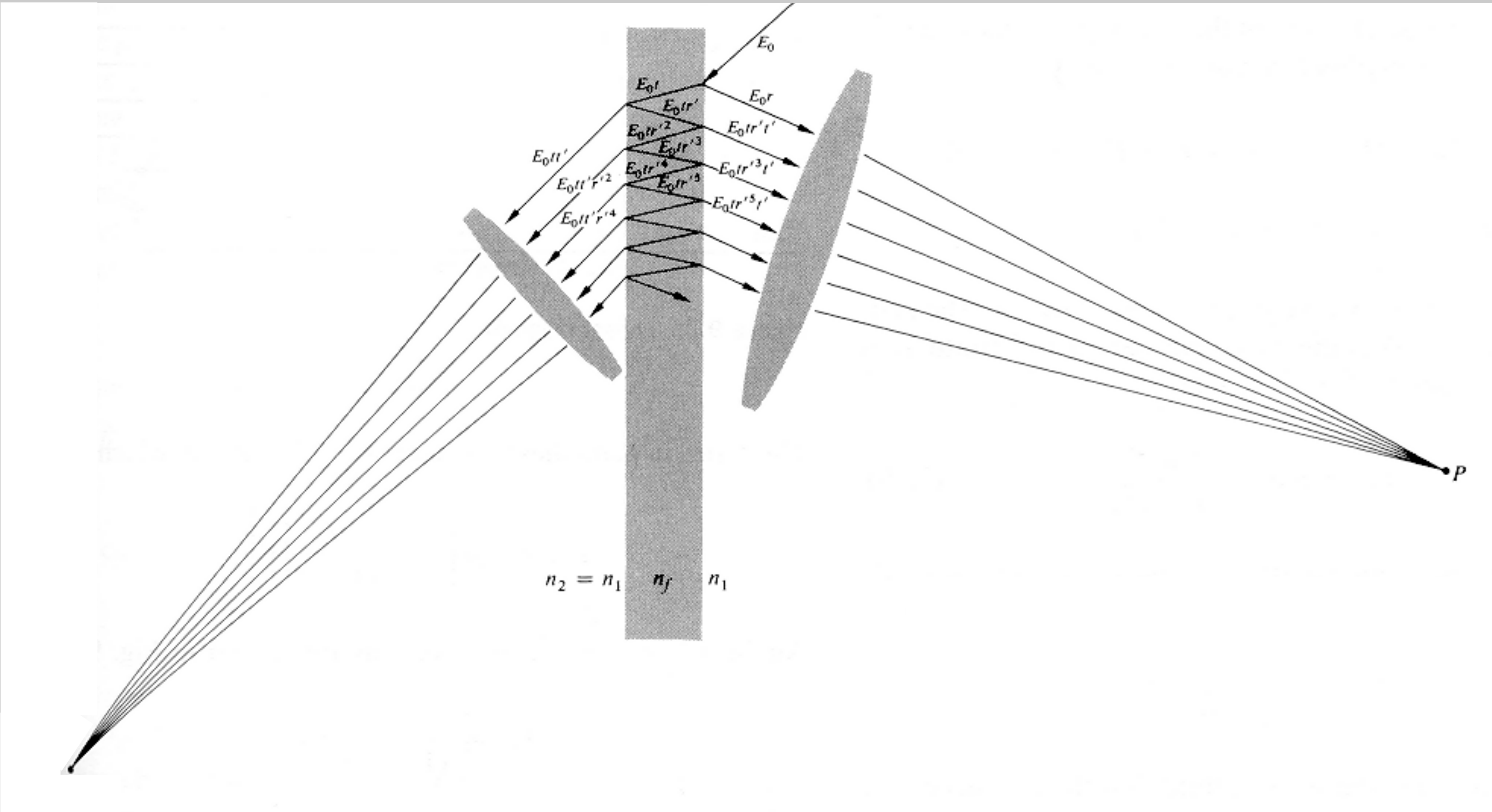
- This approach can be used to calculate phase of prism pairs and grating pairs



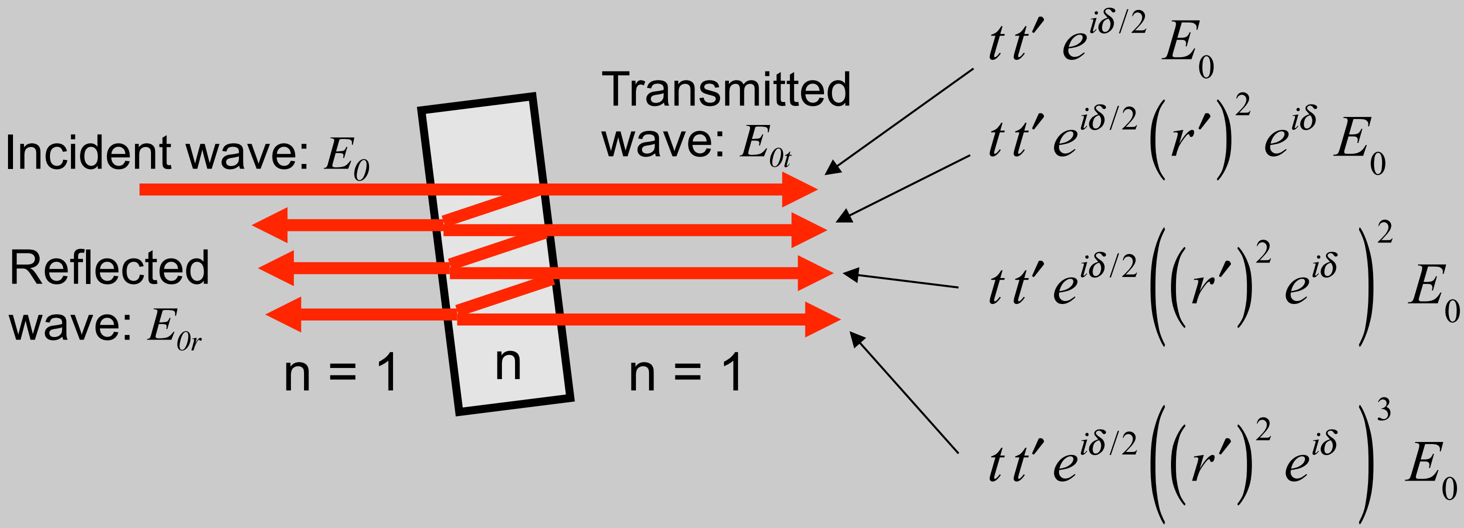
Multiple-beam interference: The Fabry-Perot Interferometer or Etalon

A Fabry-Perot interferometer is a pair of **parallel** surfaces that reflect beams back and forth. An etalon is a type of Fabry-Perot etalon, and is a piece of glass with parallel sides.

The transmitted wave is an infinite series of multiply reflected beams.



Multiple-beam interference: general formulation



r, t = reflection, transmission coefficients from air to glass

r', t' = " " " from glass to air

δ = round-trip phase delay inside medium = $k_0(2\ n\ L\cos\theta_t)$

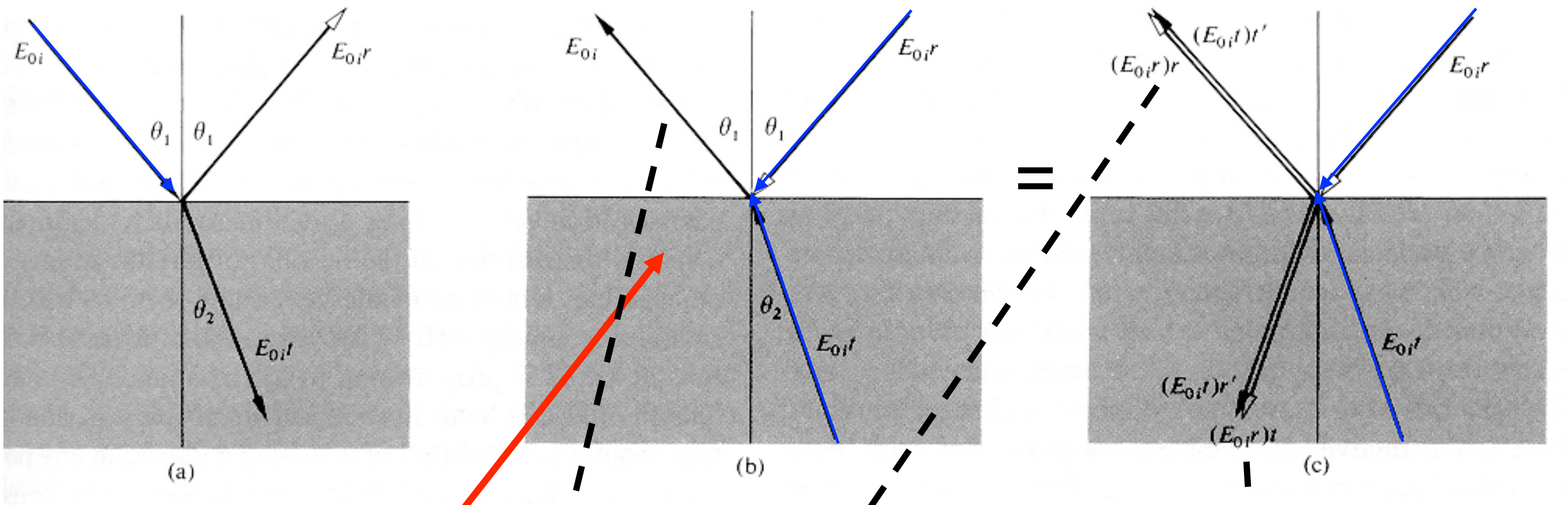
Transmitted wave:

$$E_{0t} = tt'e^{-i\delta/2}E_0\left(1+\left(r'\right)^2 e^{i\delta}+\left(\left(r'\right)^2 e^{i\delta}\right)^2+\left(\left(r'\right)^2 e^{i\delta}\right)^3+\ldots\right)$$

Reflected wave:

$$E_{0r} = rE_0 + tt'r'e^{i\delta}E_0 + tt'r'\left(\left(r'\right)^2 e^{i\delta}\right)^2 E_0 + \ldots$$

Stokes Relations for reflection and transmission



"Time reversal:"
Same amplitudes, reversed propagation direction

$$E_{oi} = (E_{oi}r)r + (E_{oi}t)t'$$
$$\therefore tt' = 1 - r^2$$

$$(E_{oi}t)r' + (E_{oi}r)t = 0$$
$$\therefore r' = -r$$

Notes:

- **relations apply to angles connected by Snell's Law**
- **true for any polarization, but not TIR**
- **convention for which interface experiences a sign change can vary**

Fabry-Perot transmission

Stokes' relations $r' = -r$, $r'^2 = r^2$, $tt' = 1 - r^2$

The transmitted wave field is:

$$E_{0t} = tt' e^{i\delta/2} E_0 \left(1 + (r')^2 e^{i\delta} + \left((r')^2 e^{i\delta}\right)^2 + \left((r')^2 e^{i\delta}\right)^3 + \ldots\right)$$

$$= tt' e^{i\delta/2} E_0 \left(1 + r^2 e^{i\delta} + \left(r^2 e^{i\delta}\right)^2 + \left(r^2 e^{i\delta}\right)^3 + \ldots\right) = tt' e^{i\delta/2} E_0 \sum_{n=0}^{\infty} \left(r^2 e^{i\delta}\right)^n$$

Where:

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots$$

$$\Rightarrow \quad \boxed{E_{0t} = \frac{tt' e^{i\delta/2}}{1 - r^2 e^{-i\delta}} E_0}$$

Power transmittance: $T \equiv \left|\frac{E_{0t}}{E_0}\right|^2 = \left|\frac{tt' e^{i\delta/2}}{1 - r^2 e^{i\delta}}\right|^2 = \frac{(tt')^2}{(1 - r^2 e^{+i\delta})(1 - r^2 e^{-i\delta})}$

$$= \left[\frac{(tt')^2}{\{1 + r^4 - 2r^2 \cos\delta\}}\right] = \left[\frac{(1-r^2)^2}{\{1 + r^4 - 2r^2[1 - 2\sin^2(\delta/2)]\}}\right] = \left[\frac{(1-r^2)^2}{\{1 - 2r^2 + r^4 + 4r^2 \sin^2(\delta/2)]\}}\right]$$

Dividing numerator and denominator by $(1-r^2)^2$

$$\boxed{T = \frac{1}{1 + F\sin^2(\delta/2)}}$$

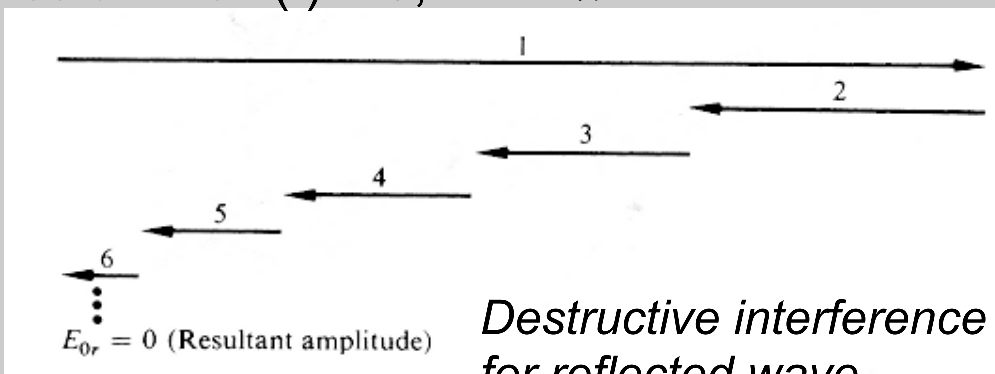
where: $F = \left[\frac{2r}{1-r^2}\right]^2$

Multiple-beam interference: simple limits

Reflected waves

$$T = \frac{1}{1 + F\sin^2\left(\delta/2\right)}$$

Full transmission: sin( ) = 0, d = 2 π m

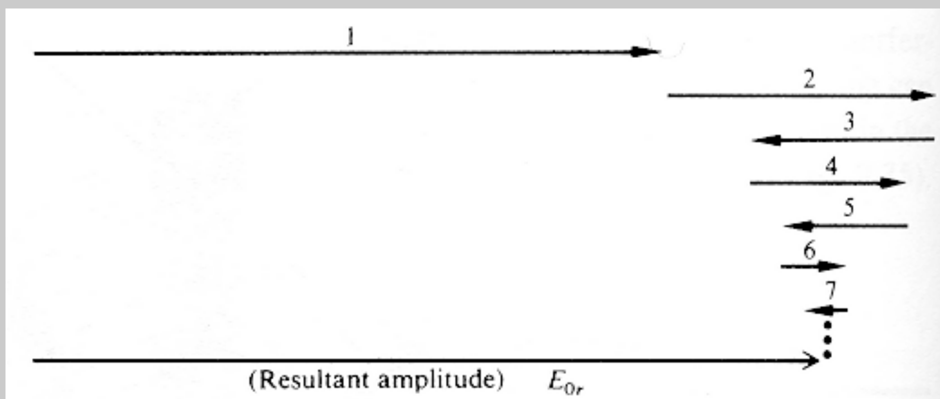


1st reflection

} internal reflections

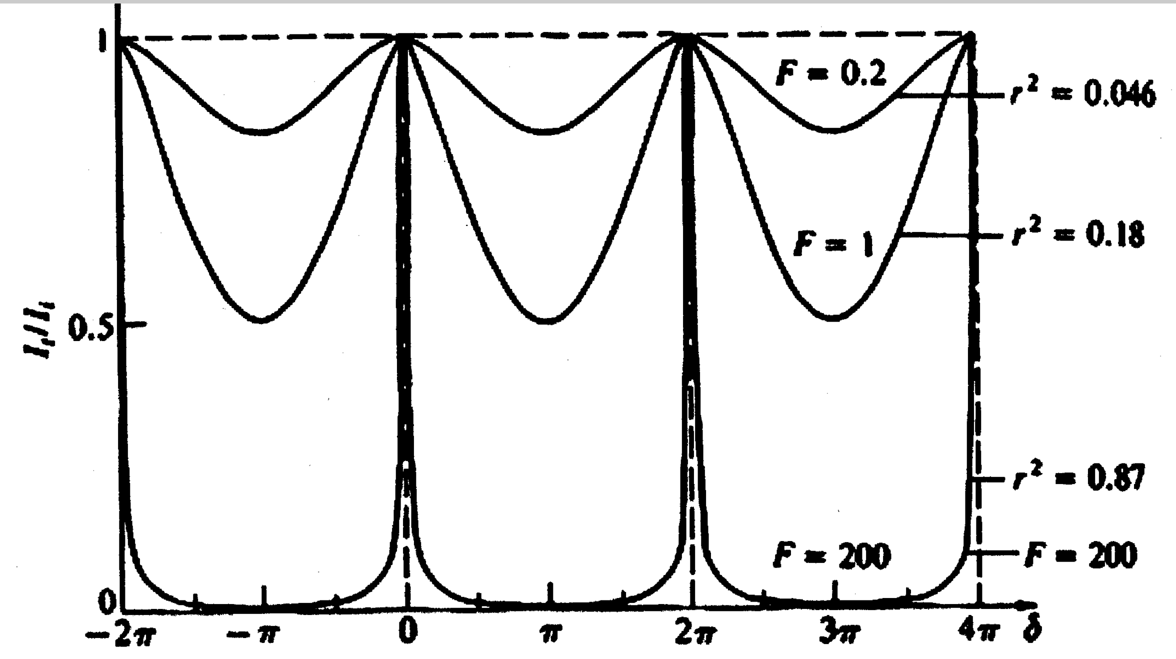
Destructive interference for reflected wave

Minimum transmission: sin( ) = 1, d = 2 π (m+1/2)



Constructive interference for reflected wave

Etalon transmittance vs. thickness, wavelength, or angle



$$T = \frac{1}{1 + F\sin^2(\delta/2)}$$

Transmission max:
sin( ) = 0, d = 2 π m

$$\delta = \frac{\omega}{c} 2nL\cos[\theta_t]$$
$$= 2\pi m$$

At normal incidence:

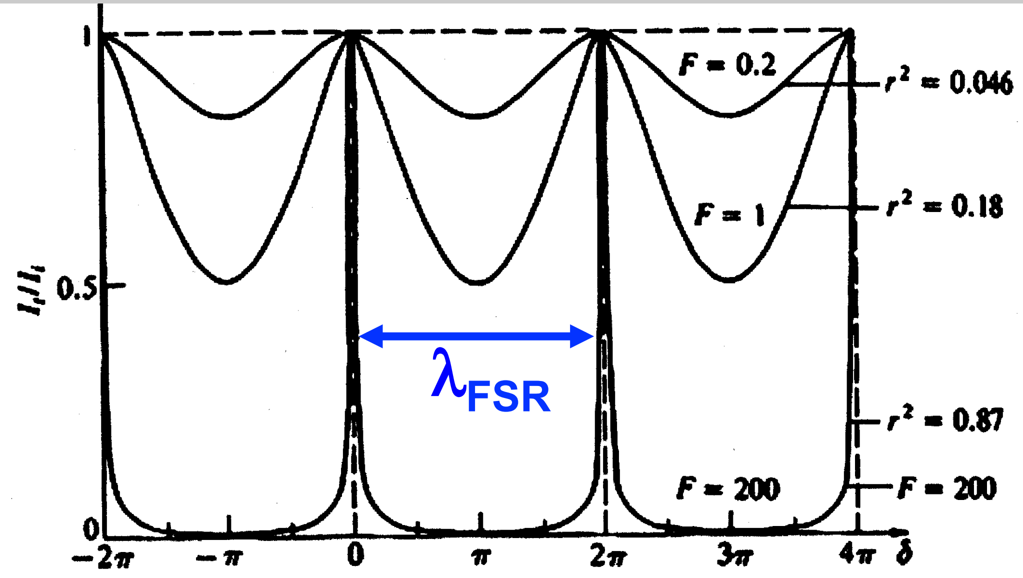
$$\lambda_m = \frac{2nL}{m} \quad \text{or} \quad nL = m\frac{\lambda_m}{2}$$

- The transmittance varies significantly with thickness or wavelength.
- We can also vary the incidence angle, which also affects δ.
- As the reflectance of each surface ($R=r^2$) approaches 1, the widths of the high-transmission regions become very narrow.

The Etalon Free Spectral Range

The Free Spectral Range is the wavelength range between transmission maxima.

λ_{FSR} = Free Spectral Range



For neighboring orders:

$$\frac{4\pi nL}{\lambda_1} - \frac{4\pi nL}{\lambda_2} = 2\pi \Rightarrow \frac{1}{\lambda_1} - \frac{1}{\lambda_2} = \frac{1}{2nL} = \frac{\lambda_2 - \lambda_1}{\lambda_1 \lambda_2}$$

$$\lambda_2 - \lambda_1 = \lambda_{FSR}$$

$$\lambda_2 \lambda_1 \approx \lambda^2$$

$$\lambda_{FSR} \approx \frac{\lambda^2}{2nL}$$

$$\frac{\lambda_{FSR}}{\lambda} = \frac{\lambda}{2nL} = \frac{\nu_{FSR}}{\nu}$$

$$\nu_{FSR} \approx \frac{c}{2nL}$$

1/(round trip time)

Etalon Linewidth

The Linewidth δ_{LW} is a transmittance peak's full-width-half-max (FWHM).

$$T = \frac{1}{1 + F\sin^2(\delta/2)}$$

A maximum is where $\delta/2 \approx m\pi + \delta'/2$ and $\sin^2(\delta/2) \approx \delta'/2$

Under these conditions (near resonance),

$$T = \frac{1}{1 + F\delta'^2/4}$$

This is a Lorentzian profile, with FWHM at:

$$\frac{F}{4}\left(\frac{\delta_{LW}}{2}\right)^2 = 1 \quad \Rightarrow \quad \delta_{LW} \approx 4/\sqrt{F}$$

This transmission linewidth corresponds to the minimum resolvable wavelength.

Etalon Finesse ≈ resolution

The Finesse, $\Im$, is the ratio of the Free Spectral Range and the Linewidth:

$$\Im \equiv \frac{\delta_{FSR}}{\delta_{FW}} = \frac{2\pi}{4/\sqrt{F}} = \frac{\pi\sqrt{F}}{2}$$

δ = 2π corresponds to one FSR

Using: $$F = \left[\frac{2r}{1-r^2}\right]^2$$

$$\Im = \frac{\pi}{1-r^2}$$

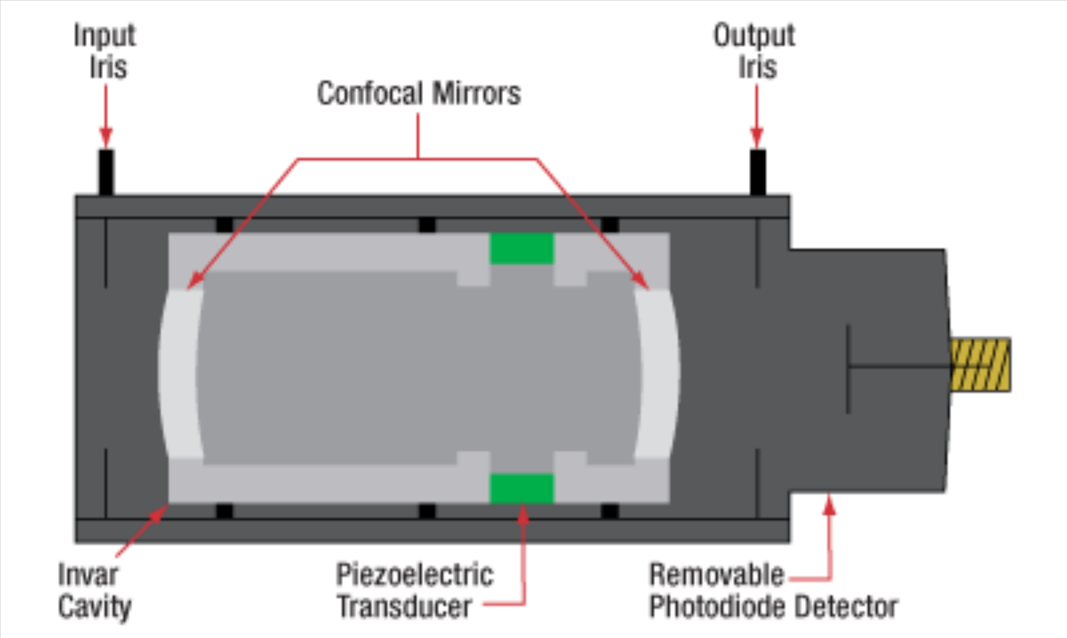
taking $r \approx 1$

The Finesse is the number of wavelengths the interferometer can resolve.

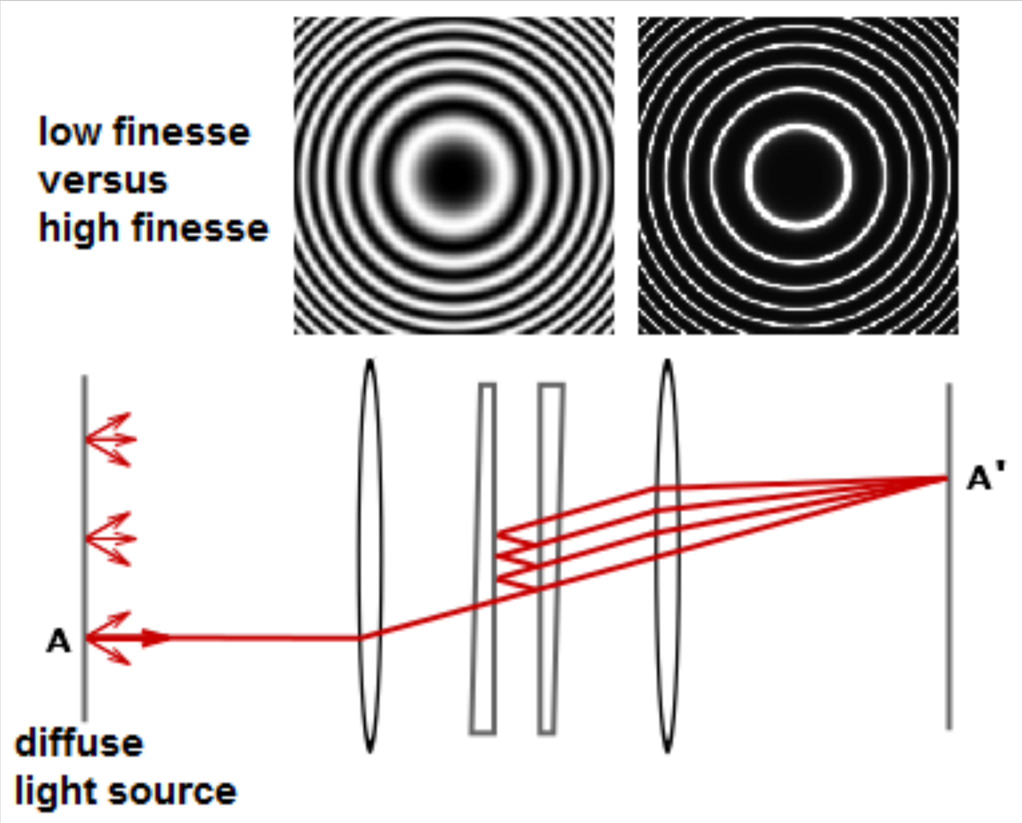
Tools: scanning Fabry-Perot

Resonator with piezo control over mirror separation
http://www.thorlabs.us/newgrouppage9.cfm?objectgroup_id=859

- Wavelength range: 535-820nm (ours)
- SA200 (ours)
  - FSR 1.5 GHz
  - Finesse > 200
  - Resolution 7.5MHz
- SA210
  - FSR 10 GHz
  - Finesse > 150
  - Resolution 67MHz



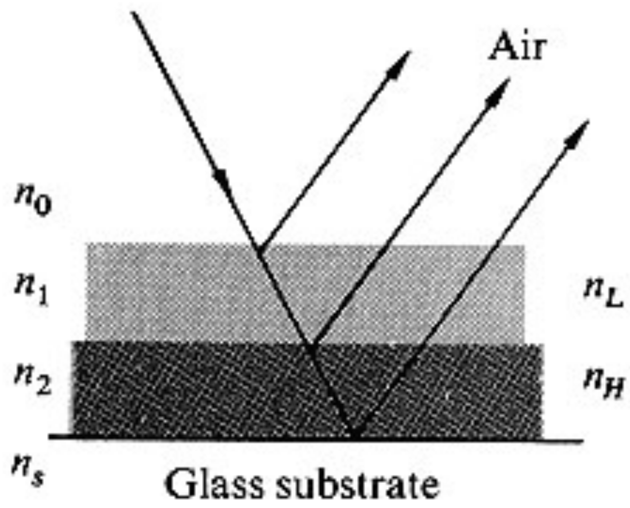
Tools: fixed plate Fabry-Perot



Multilayer coatings

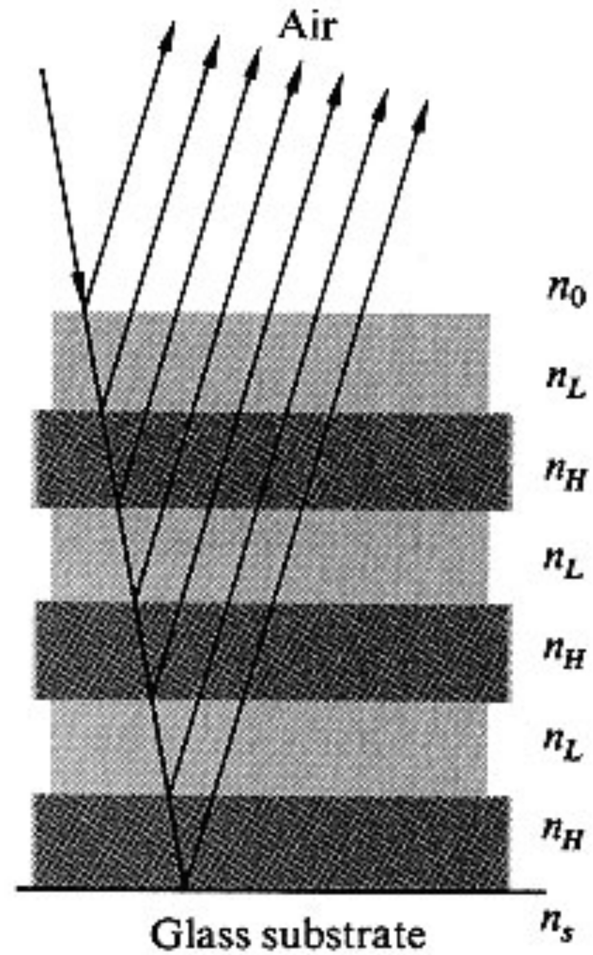
Typical laser mirrors and camera lenses use many layers.

The reflectance and transmittance can be custom designed



$g\,HL\,a$

Double-quarter

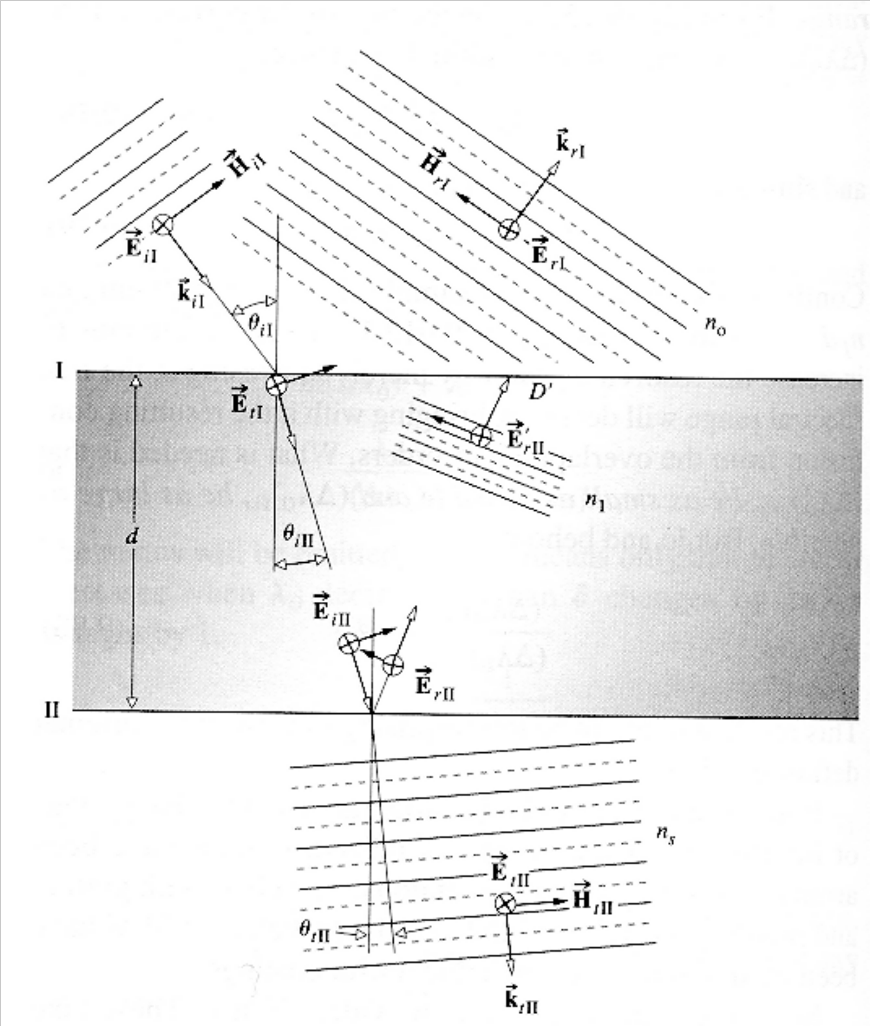


$g\,HL\,HL\,HL\,a$

$g(HL)^3a$

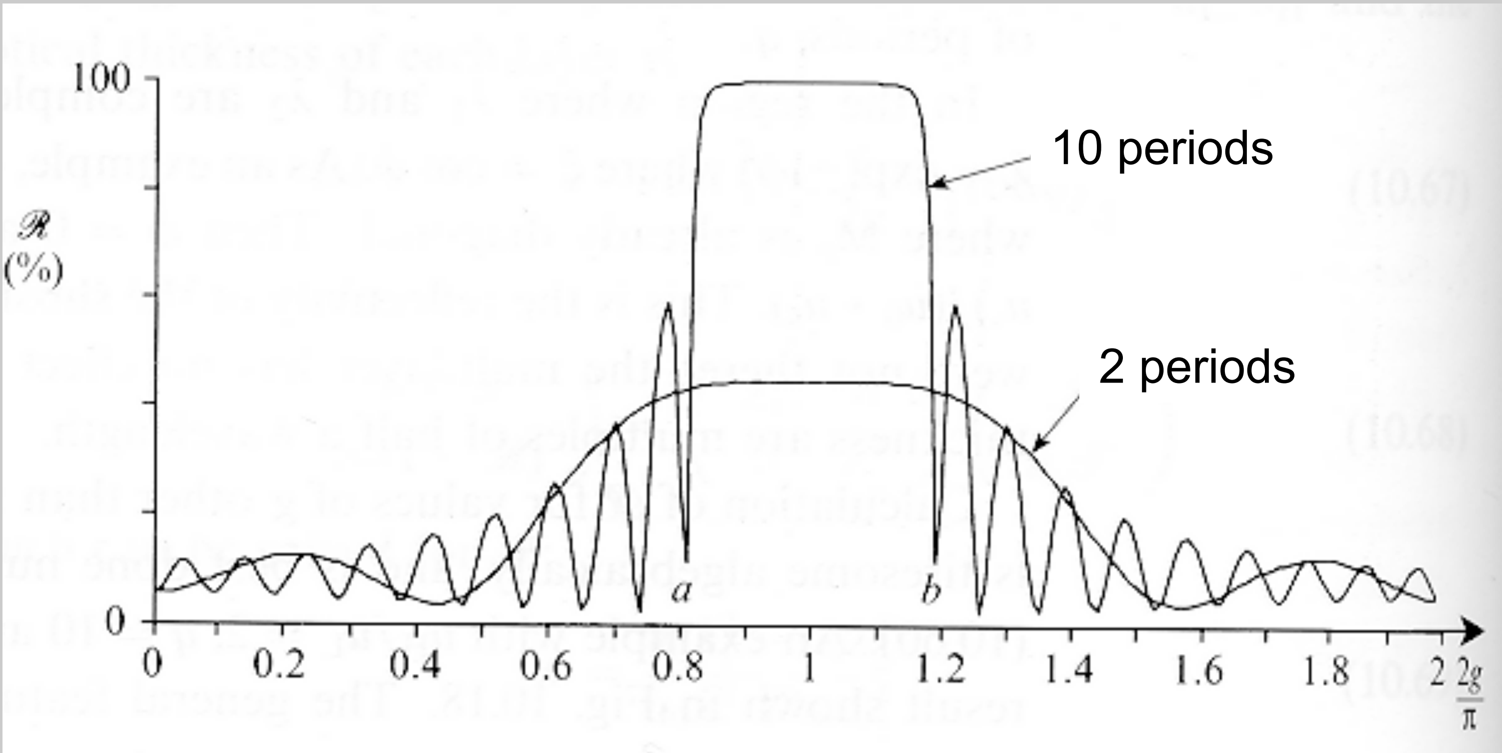
Quarter-wave stack

Multilayer thin-films: wave/matrix treatment



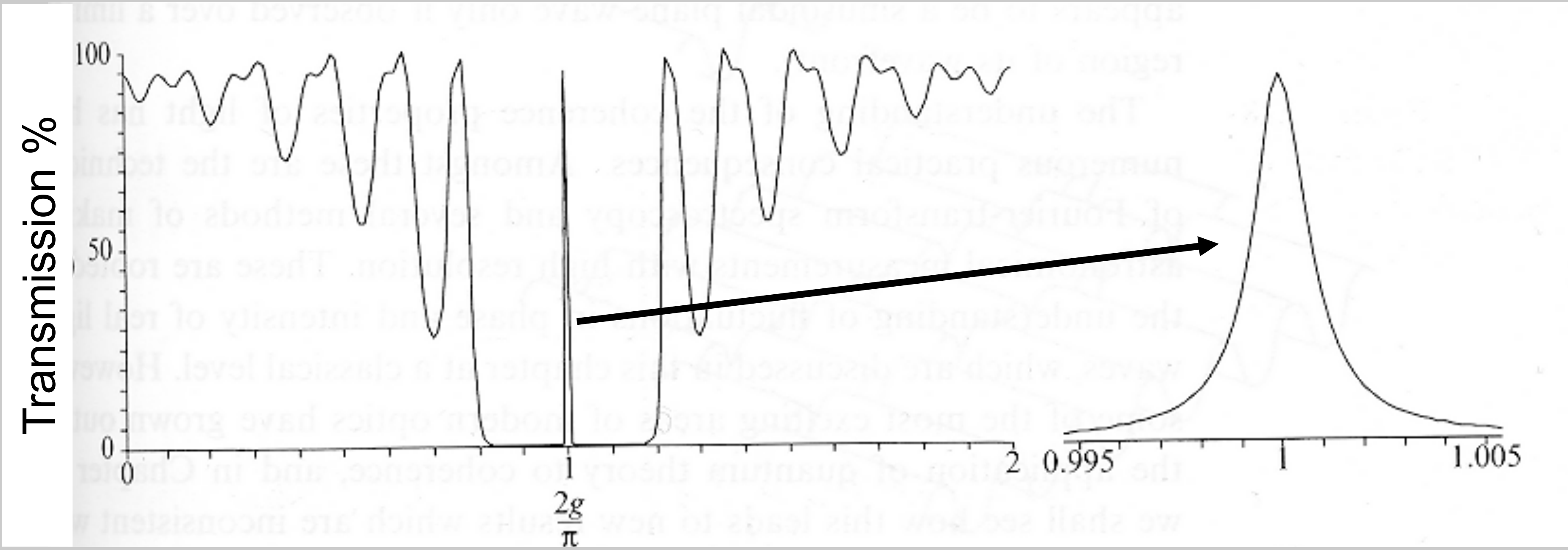
- Use boundary conditions to relate fields at the boundaries
- Phase shifts connect fields just after I to fields just before II
- Express this relation as a transfer matrix
- Multiply matrices for multiple layers

High-reflector design



Reflectivity can reach > 99.99% at a specific wavelength
> 99.5% for over 250nm
Bandwidth and reflectivity are better for “S” polarization.

Interference filter design



A thin layer is sandwiched between two high reflector coatings

-very large free spectral range, high finesse

- typically 5-10nm bandwidth, available throughout UV to IR