PHGN 462 Homework 6

1) Pollack and Stump 14.1. For those of you interested in the whole group vs. phase velocity thing, this problem will show quite explicitly using the Poynting vector that the energy transport in this particular context is definitely associated with the *group* velocity.

2) Pollack and Stump 14.6. Short, plug 'n chug. Don't overdo it.

3) We're spending a lot of time talking about how E&M waves propagate in various waveguides, and not a lot of time thinking about why we bother to propagate them. Root around in books or on the interwebs and learn about the actual applications of microwave waveguides. Write up a short description of two different applications. It would be nice if at least one of them came into common usage after 1960 (a lot of microwave tech is pretty old).

And don't take advantage of this problem by copying and pasting stuff from, say, the first and second websites that come up when you search for "microwave waveguides." I know which sites those are. I also know what the third and fourth hits are, before you get any clever ideas.

4) I remember back in 361 when we were talking about hyperbolic trig functions I tried to really hammer home the point that special functions aren't all that special. At least, they don't need to be scary. I think I said something like "Even Bessel functions aren't too bad as long as you stay calm and look up their properties."

Anyway, let's think about a hollow conducting cylindrical waveguide of radius a (this is not the same as the coaxial pair of cylindrical shells that we did/will do in class… just one cylinder). Let the axis of the cylinder be in the $\hat{k}$ direction.

a) There are a lot of different ways to construct solutions to a new geometry. But easiest is to take a look at the general problem that isn't specific to any geometry. Section 14.3 does this, so read that up until eqn 14.74. That equation should look pretty familiar, because we've done this before: We solved the general problem and then made it a specific problem by applying rectangular boundary conditions. But this time we're going to apply cylindrical boundary conditions, so solve 14.74 using separation of variables in cylindrical coordinates (technically polar since we're in 2D).

b) If you start going after the above with separation of variables, you should end up with a fairly trivial equation for φ and Bessel's equation for γr, where $\gamma^2 = \left(\frac{\omega}{c}\right)^2 - k^2$. Don't panic! Just read about Bessel's equation and the solutions to it. We're not really going to have to do all that much with them. Put everything together and write down the solution for ψ and then for the magnetic field. The solution should be indexed by some integer; let's call it m. Use a complex exponential for the φ equation instead of sines and cosines.

c) Demonstrate how to find the parameter γ corresponding to a particular mode m. With that, you can complete the dispersion relation. Unless you're very clever, you'll probably at some point have to say "And here's where we can't proceed analytically anymore, so numbers." And that's fine. Sometimes that really is the answer.

5)

a) Pollack and Stump 14.8. If you set it up right you can make it look pretty darn similar to normal incidence reflection as we've done it in the past and have very little actual work left to do. Just something to consider.

b) When we found reflection and transmission coefficients in the free-space normal incidence situation, were R & T functions of frequency? That is, did different colors reflect differently? What about now, doing the same thing in a waveguide?