$$\vec{E} = \int \frac{1}{4\pi\epsilon_0} \frac{\rho\, d\tau'}{\gamma^2} \hat{\gamma}$$

$$\_\_\ \hat{r} + \_\_\ \hat{\theta} + \_\_\ \hat{\varphi}$$

we always put $\hat{\gamma} = \_\_\ \hat{x} + \_\_\ \hat{y} + \_\_\ \hat{z}$

even though $d\tau' = r'^2 \sin^2\theta'\, d\theta'\, d\varphi'\, dr'$

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\rho\, d\tau'}{\gamma}$$

$$|\vec{\gamma}| = \left\{ (x - r'\sin\theta'\cos\varphi')^2 + (y - r'\sin\theta'\sin\varphi')^2 + z^2 \right\}^{1/2}$$

$$\vec{E} = -\vec{\nabla} V$$

& other coords

Sep Variables for Laplace's eqn

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \phi$$

← PDE ← partial

Assume $V(x, y, z) = X(x)\, Y(y)\, Z(z)$

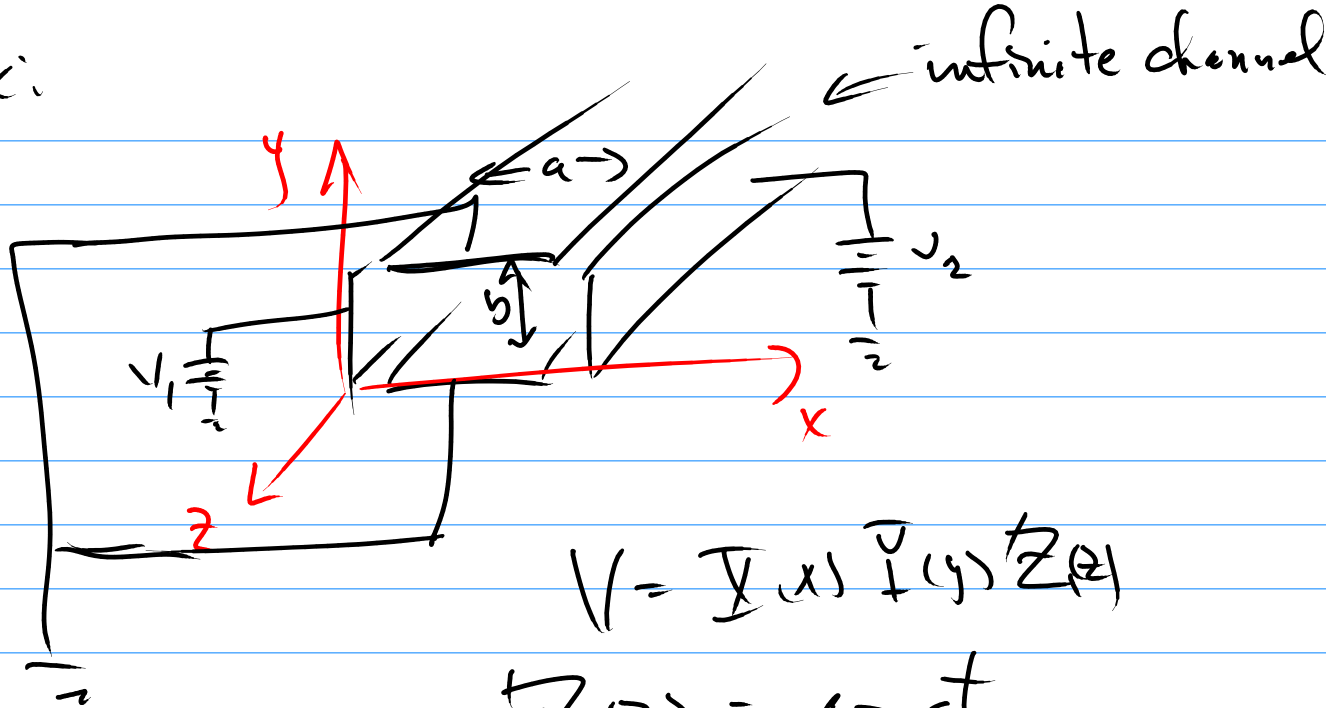
Plug soln into $\nabla^2 V = 0$ ÷

divide by $V = XYZ$ gives

$$\underbrace{\frac{1}{X}\frac{d^2 X}{dx^2}}_{C_1} + \underbrace{\frac{1}{Y}\frac{d^2 Y}{dy^2}}_{C_2} + \underbrace{\frac{1}{Z}\frac{d^2 Z}{dz^2}}_{C_2} = 0$$

function of x ↑ function of y ↑ ← ODE ← ordinary

Ex:



$$V = X(x)\, Y(y)\, Z(z)$$

$$Z(z) = \text{const}$$

$$C_3 = \frac{1}{Z}\frac{d^2 Z}{dz^2} = 0$$

$$C_1 + C_2 = 0$$

Choose y direction (grounded at both ends)

$$C_2 = \frac{1}{Y}\frac{d^2 Y}{dy^2}$$

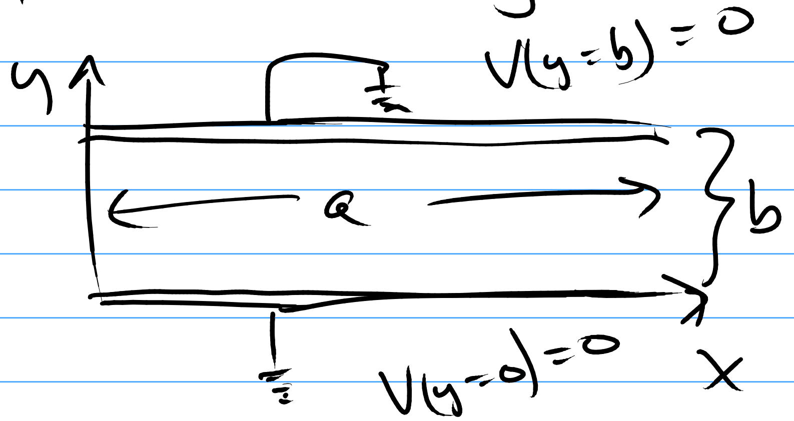
Solns are

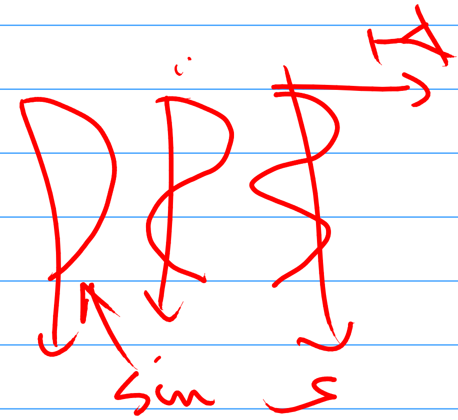
$$\frac{d^2 Y}{dy^2} = C_2 Y \qquad \text{exp, sin cos,}$$

$$Y = A \sin ky + B \cos ky \qquad C_2 < 0$$

$$Y = A' e^{\sqrt{c_2}\, y} + B' e^{-\sqrt{c_2}\, y} \qquad C_2 > 0$$

$$Y = A'' + B'' y \qquad C_2 = 0$$





$$Y = A \sin kb \qquad k = \frac{n\pi}{b} \qquad n = 1, 2, 3, \ldots$$

$$Y(y=0) = 0 \qquad Y(y=b) = 0$$

$$C_1 + C_2 + C_3 = 0 \Rightarrow C_1 = k^2 > 0$$

$$C_1 - k^2 + 0$$

$$\overline{X}(x) = Ge^{kx} + He^{-kx}$$

$$V = \overline{X}(x)\,\overset{\circ}{\bot}\,Z_4 = \left(Ge^{kx} + He^{-kx}\right)\left(A\sin ky\right)\,e$$