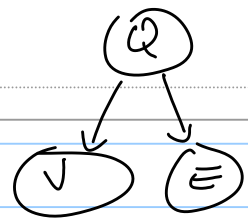
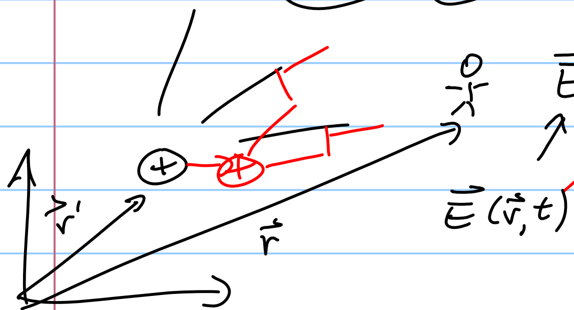
Note Title 6/28/2006



$$\vec{E} = \int \frac{k\,dQ}{\mathscr{r}^2}\,\hat{\mathscr{r}} = \int \frac{k\,\rho\,d\tau}{\mathscr{r}^2}\,\hat{\mathscr{r}}$$



$$\cancel{\vec{E} = k\int \frac{\rho(\vec{r}',t_r)}{\mathscr{r}^2}\,d\tau'\,\hat{\mathscr{r}}}$$

wrong from observation

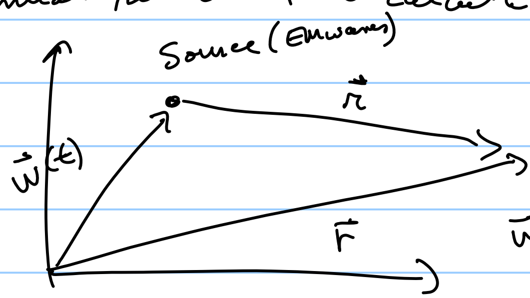
$$t_r = t - \frac{\mathscr{r}}{c}$$

special: only when source & receiver are fixed

What matches expt

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}',t_r)}{\mathscr{r}}\,d\tau' \qquad V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}',t_r)}{\mathscr{r}}\,d\tau'$$

general retarded time calculations



distance $|\vec{\mathscr{r}}| = |\vec{r} - \vec{w}|$

$\vec{w} + \vec{\mathscr{r}} = \vec{r}$

at t_r light pulse is emitted and distance pulse has to travel is $|\vec{r} - \vec{w}(t_r)| = c(t - t_r)$

$t_r = t - \frac{\text{dist}}{c}$

ex: fix w to be L $\quad \frac{|\vec{r} - \vec{L}|}{c} = t - t_r \quad t_r = t - \frac{|\vec{r} - \vec{L}|}{c}$

$$|\vec{r} - \vec{w}(t_r)| = c(t - t_r)$$

$$|0 - x_0 - v_0 t_r| = x_0 + v_0 t_r = ct - ct_r$$

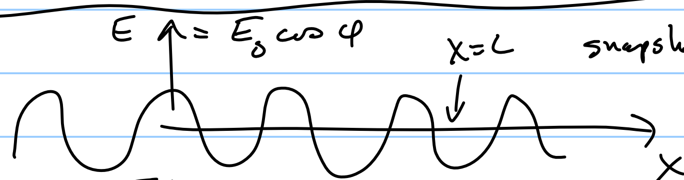
$$v_0 t_r + c t_r = ct - x_0$$

$$t_r(v_0 + c) = ct - x_0 \Rightarrow t_r = \frac{ct - x_0}{v_0 + c}$$

$v_0 \to \phi \quad t_r = \frac{t - \frac{x_0}{c}}{1 + \cancel{v_0/c}} = t - \frac{x_0}{c}$

Ex:

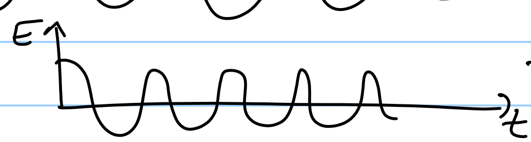
$E = E_0 \cos\varphi$



snapshot at $t=0$

fix $x=0$

$\varphi_{x=\phi} = -\omega t$ given



Know $\varphi_{x=0} = -\omega t$ $\qquad$ $\varphi(x,t) = \varphi(x=0, t_r)$

$\uparrow$ want

$$\varphi(x=0, t_r) = -\omega t_r = -\omega\left(t - \frac{x}{c}\right) = -\omega t + \underbrace{\frac{\omega}{c} x}_{kx}$$

$\lambda \nu = c$ $\qquad$ $k = \frac{2\pi}{\lambda}$ $\qquad$ $\omega = 2\pi\nu$ $\qquad$ $\frac{\omega}{c} = \frac{2\pi\nu}{c} = \frac{2\pi}{\lambda}$

$\frac{\nu}{c} = \frac{1}{\lambda}$

$$\vec{E}(x,t) = \vec{E}_0 \cos(kx - \omega t)$$

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \qquad \vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,', t_r)}{\mathscr{r}} d\tau' \qquad V(\vec{r},t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,', t_r)}{\mathscr{r}} d\tau'$$

$\vec{\nabla}$ $V(x,y,z,t)$

$$\vec{\nabla} V = \vec{\nabla} \frac{1}{4\pi\epsilon_0} \int \left(\frac{\rho(\vec{r}\,', t_r)}{\mathscr{r}}\right) d\tau'$$

$\uparrow$

unprimed variables

$$t_r = t - \frac{\mathscr{r}}{c}$$

$$\int_0^L x' dx = \frac{L^2}{2}$$

$$\vec{\nabla}\left(\frac{\rho}{r}\right) = \rho\,\vec{\nabla}\left(\frac{1}{r}\right) + \frac{1}{r}\vec{\nabla}\rho$$

$$\vec{\nabla}\left(\frac{1}{r}\right) = -\frac{\hat{r}}{r^2} \quad \text{eqn } 1.81$$

$$V_{pt} \propto \frac{1}{r}$$

$$\vec{E} = -\vec{\nabla}V \propto -\vec{\nabla}\left(\frac{1}{r}\right) \propto \frac{\hat{r}}{r^2}$$

$$\vec{\nabla}\rho(\vec{r}', t_r) = \hat{x}\frac{\partial\rho}{\partial x} + \hat{y}\frac{\partial\rho}{\partial y} + \hat{z}\frac{\partial\rho}{\partial z}$$

$$t_r = t - \frac{r}{c}$$

$$\frac{\partial\rho}{\partial x} = \frac{\partial\rho}{\partial t_r}\frac{\partial t_r}{\partial x}$$

$$\frac{\partial\rho}{\partial t_r} \leftrightarrow \frac{\partial\rho}{\partial t}$$

$$\frac{dt_r}{dt} = 1 \qquad dt_r = dt$$

$$\vec{\nabla}\rho = \dot{\rho}\underbrace{\left(\hat{x}\frac{\partial t_r}{\partial x} + \hat{y}\frac{\partial t_r}{\partial y} + \hat{z}\frac{\partial t_r}{\partial z}\right)}_{\vec{\nabla}t_r} = \dot{\rho}\,\vec{\nabla}t_r$$

$$\vec{\nabla}t_r = \vec{\nabla}\left(t - \frac{r}{c}\right) = -\frac{1}{c}\vec{\nabla}r$$

$$(\vec{\nabla}r)_x = \frac{\partial}{\partial x}\sqrt{(x-x')^2+(y-y')^2+(z-z')^2} = \frac{1}{\cancel{2}}\frac{1}{r}\cancel{2}(x-x')$$

$$\vec{\nabla} \mathscr{r} = \frac{\vec{\mathscr{r}}}{\mathscr{r}} = \hat{\mathscr{r}} \qquad \text{So} \qquad \vec{\nabla} t_r = -\frac{\hat{\mathscr{r}}}{c}$$

$$\vec{\nabla} V = -\frac{1}{4\pi\epsilon_0} \int \left\{ \rho(\vec{r}', t_r)\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} + \frac{\dot{\rho}(\vec{r}', t_r)}{\mathscr{r}} \frac{\hat{\mathscr{r}}}{c} \right\} d\tau'$$

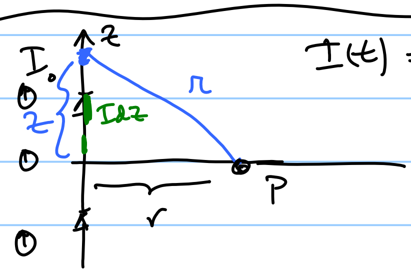
$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} = \underline{\text{Jefimenko's eqn}}$$

Sec. 10.2.1 : take V & A with retarded times and show that ME are obeyed. Start with $\vec{A}$ & V and then derive PDE's for A & V

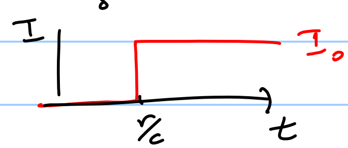
ME in terms of V & $\vec{A}$

$$\nabla^2 V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0} \qquad \nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J}$$

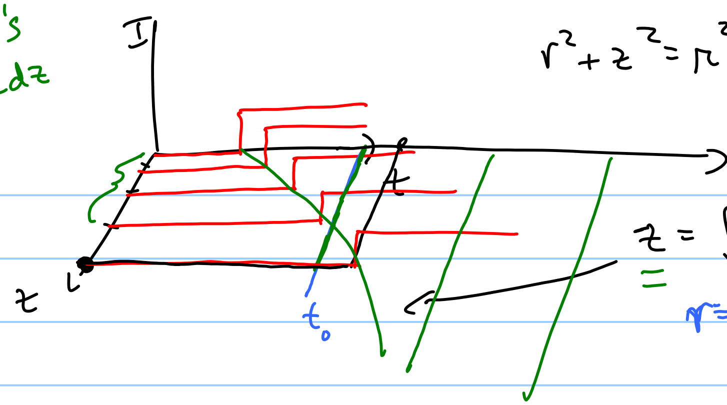
Ex: 10.2



$$I(t) = \begin{cases} 0 & t \leq 0 \\ I_0 & t > 0 \end{cases}$$



Sum $d\vec{A}'s$ due to $I\,dz$



$$r^2 + z^2 = \mathscr{r}^2 = (ct)^2$$

$$z = \sqrt{c^2t^2 - r^2}$$

$r = 0 \quad z = ct$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,', t_r)}{\mathscr{r}}\, d\tau' = \frac{\mu_0}{4\pi}\int \frac{I(\vec{r}\,', t_r)\, d\vec{l}}{\mathscr{r}}$$

I_0 is constant $\qquad d\vec{l} = dz\,\hat{z}$

$$\vec{A} = \frac{\mu_0 \hat{z}}{4\pi}\int \frac{I_0\, dz}{\mathscr{r}} \qquad \vec{A} = \int_{-\sqrt{c^2t^2-r^2}}^{\sqrt{c^2t^2-r^2}} d\vec{A}$$