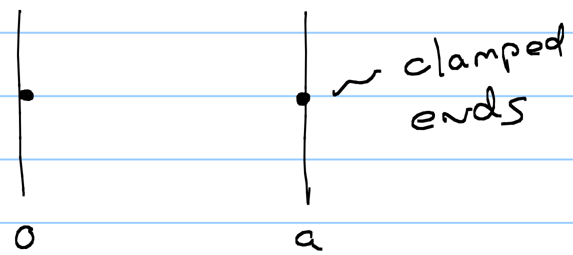
Note Title 2/29/2008

Classical vs Quantum

(1C) $$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

(1Q) $$\frac{\partial^2 \psi}{\partial x^2} = -i\,\frac{2m}{\hbar}\,\frac{\partial \psi}{\partial t}$$



$u \propto \cos \omega t$ — const. freq. soln — normal mode

$\psi \propto e^{-iE/\hbar\, t}$ — const. energy soln — stationary state

$$u(x,t) = A \sin\left(\frac{N\pi x}{a}\right)\cos \omega t$$

$$\psi(x,t) = B \sin\left(\frac{N\pi x}{a}\right) e^{-iEt/\hbar}$$

solution of (1C) $\Rightarrow$
$$\frac{\omega^2}{c^2} = \left(\frac{N\pi}{a}\right)^2$$

solution of (1Q) $\Rightarrow$
$$\frac{2mE}{\hbar^2} = \left(\frac{N\pi}{a}\right)^2$$

$$\boxed{\omega = \frac{N\pi c}{a}}$$

$$\boxed{E = \frac{(N\pi\hbar)^2}{2ma^2}}$$

The strings energy depends on A.

$$E = \frac{\rho N^2 \pi^2 c^2 A^2}{4a}$$

$$\int_0^a \frac{1}{2}\rho |\dot{u}|^2 \, dx$$

$$\dot{u} = -\omega A \sin\left(\frac{n\pi x}{a}\right) \sin\omega t$$

$$\int_0^a \rho \frac{\omega^2}{2} A^2 \overline{\sin^2 \omega t} \; \overline{\sin^2\left(\frac{n\pi x}{a}\right)} \, dx$$

$$\frac{a}{2}\left(\frac{n\pi c}{a}\right)^2 A^2 \quad 1 \qquad \text{avg. over 1 period}$$

$$E = \rho \frac{n^2 \pi^2 c^2 A^2}{4a}$$

Recall in the photoelectric effect: energy of ejected electrons is independent of amplitude of incident light waves.

Sample question. For an electron confined to a well of atomic dim say, $a = .1\,nm = 10^{-10}\,m$

Compute (estimate) the first 3 energy levels.

$$E = \frac{(n\pi\hbar)^2}{2ma^2}$$

$\hbar = 1 \times 10^{-34}$ JS

$m = 9 \times 10^{-31}$ Kg

$$\frac{\pi^2\hbar^2}{2ma^2} = \frac{9 \cdot 1 \times 10^{-68}}{2 \cdot 9 \times 10^{-31} \cdot 10^{-20}}$$

$$= \frac{1}{2} \cdot 10^{-17} \text{ J}$$

$1 \text{ J} = 6 \times 10^{18}$ eV

$$\Rightarrow \frac{\pi^2\hbar^2}{2ma^2} = 15 \text{ eV}$$

$E_1 = 15$

$E_2 = 4 \cdot 15 = 60$

$E_3 = 9 \cdot 15 = 135$

Sample question

If the electron makes a transition from $n=2 \to n=1$ compute the energy and wavelength of emitted photon

$\hbar\omega = 35$ eV

or $hf = 35$ eV

$$f = \frac{35 \text{ eV} / 6 \times 10^{18} \text{ eV/J}}{6 \times 10^{-34} \text{ J S}}$$

$\approx \quad 1 \times 10^{+16}$ Hz

$$\lambda = \frac{c}{f} = \frac{3\times10^{8}\ m/s}{1\times10^{16}\ 1/s} = 3\times10^{-8}\ m$$

$= 30$ nm

Sodium D Line at 189 nm

Put c, $\hbar$, J to eV
conversion on your cheat sheet.
Also $\int_{-\infty}^{\infty} e^{-x^2}\,dx \quad \int_{-\infty}^{\infty} x^2 e^{-x^2}\,dx$

$\int \sin^2(x)\,dx \quad \int \cos^2(x)\,dx$