Nonlinear Optics
Homework 4
due Monday, 7 March 2011

- **Problem 1:**
  In third-harmonic microscopy, harmonic light is generated when there is an interface in the focus. As we discussed in class, the signal is not really generated *at* the interface, but it is localized near there because of the role of phase matching and the dependence of the signal on the cube of the intensity. In this problem, we will consider the longitudinal spatial resolution of this kind of microscopy. Here are the parameters:
  - We focus a Gaussian beam with a vacuum wavelength of 1 micron at *f*/1 (the beam diameter to $1/e^2$ is equal to the focal length).
  - We are looking at a fused silica slide in water. Use the Sellmeier equations for fused silica shown below, and assume that in water n= 1.33 and we neglect dispersion in the water. You may assume that the thickness of the fused silica slide is much greater than the Rayleigh range of the focus. Be careful in how to introduce the refractive index of the slide into the equations for the focal spot size and the Rayleigh range: the wavelength that appears in these equations is its value in the medium i.e. $\lambda = \lambda_{vac} / n$.

  Calculate the third-harmonic signal vs. z-position as the focal spot is moved in the z-direction across the interface. The FWHM of this peak (full-width at half-maximum) is effectively the axial resolution of the imaging system in the longitudinal direction. We are not interested in the absolute strength of the signal, just how it varies with z. Use the absolute value of the expression 2.10.11b, paying attention to the limits of the integration.

- **dispersion for fused silica**

```
as1 = .6961663;
as2 = .4079426;
as3 = .8974794;
bs1 = .0684043^2;
bs2 = .1162414^2;
bs3 = 9.896161^2;
```

$$\text{ns}[\lambda\_] := \sqrt{\left(1 + \frac{\text{as1}\,\lambda^2}{\lambda^2 - \text{bs1}} + \frac{\text{as2}\,\lambda^2}{\lambda^2 - \text{bs2}} + \frac{\text{as3}\,\lambda^2}{\lambda^2 - \text{bs3}}\right)};$$

- **Problem 2:**
  Boyd problem 4.3

- **Problem 3:**
  Boyd problem 4.6
  By solving the heat equation in steady state, you should find that the temperature profile (and therefore the induced refractive index profile) is parabolic. This parabolic index profile acts as a lens. Calculate the phase shift after propagation through the rod, then compare your expression to that of a thin lens,
  $\phi(r) = -k\, r^2 / 2\, f$,
  thereby calculating the effective focal length of the thermally induced lens.

For typical parameters, use the parameter for fused silica (Table 4.5.1):

```
cm = 0.01 m; W = 1; Kelvin = 1;
Qin = 10 W / cm^3;
κ = 1. W / (m Kelvin);
a = 1 cm;
len = 1 cm;
dndT = 1.2 × 10^-5 / Kelvin;
```

- **Problem 4:**
  Boyd problem 4.7, parts (a) and (b).
- **Problem 5:**
  Boyd, problem 4.8
  Here you assume that the intensity dependence of the beam is unaffected by the nonlinearity (use the Gaussian beam formula for the variation of the beam size with position).