MATH 348 - Advanced Engineering Mathematics September 14, 2009

Homework 3, Fall 2009 **Due**: September 20, 2009

Linear Independence - Matrix Spaces - Vector Spaces

1. Determine the values of h for which the vectors are linearly dependent.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -5 \\ 7 \\ 8 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ h \end{bmatrix}$$

2. Given,

$$\mathbf{A} = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}.$$

   (a) Is $\mathbf{w}$ in the column space of $\mathbf{A}$? That is, does $\mathbf{w} \in \text{Col } \mathbf{A}$?

   (b) Is $\mathbf{w}$ in the null space of $\mathbf{A}$? That is, does $\mathbf{w} \in \text{Nul } \mathbf{A}$?

3. Given,

$$\mathbf{A} = \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 9 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix}.$$

   Determine:

   (a) A basis and dimension of Nul $\mathbf{A}$.

   (b) A basis and dimension of Col $\mathbf{A}$.

   (c) A basis and dimension of Row $\mathbf{A}$.

   (d) What is the Rank of $\mathbf{A}$?

4. Let,

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}.$$

   (a) How many vectors are in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

   (b) How many vectors are in Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

   (c) Is $\mathbf{w}$ is in Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

5. Given,

$$my'' + ky = 0, \quad m, k \in \mathbb{R}. \tag{1}$$

   (a) Show that $y_1(t) = \cos(\omega t)$ and $y_2(t) = \sin(\omega t)$, where $\omega = \sqrt{\frac{k}{m}}$, are solutions to the ODE.

   (b) Show that any function in Span$\{y_1, y_2\}$ is a solution to the ODE.[1]

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[1] Since we know from differential equations that the only solutions to (1) are $0, y_1, y_2$ we can conclude that the space of solutions to (1) forms a vector space whose basis is y_1 and y_2.