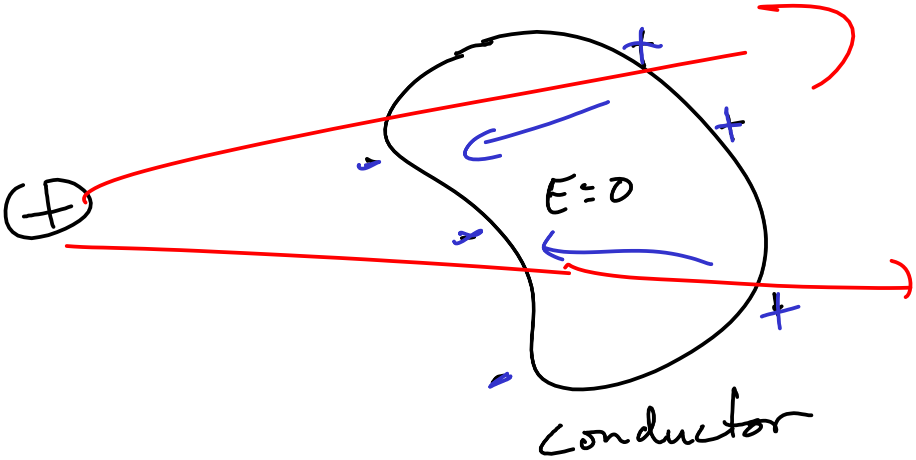
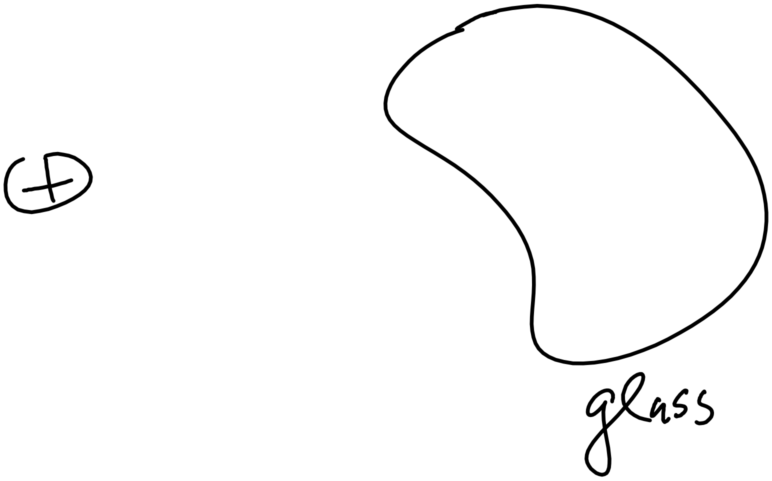
Lecture 29 Shadowitz chapter 6

Review

## Electrical properties of matter

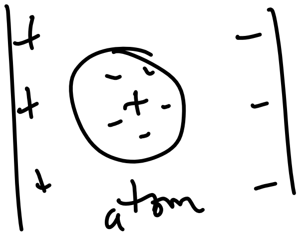




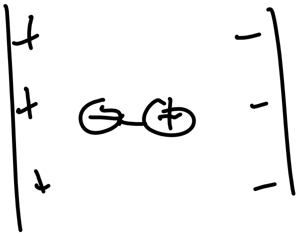
causal/creative: How can I simplify the problem to reveal the fundament physics?

Look up spherical cow on wikipedia

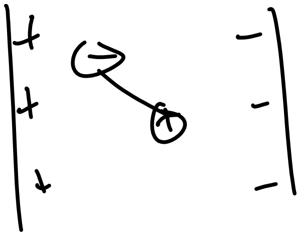
Simplify from a large number of atoms to just one.



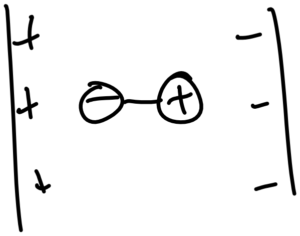
Induced dipole



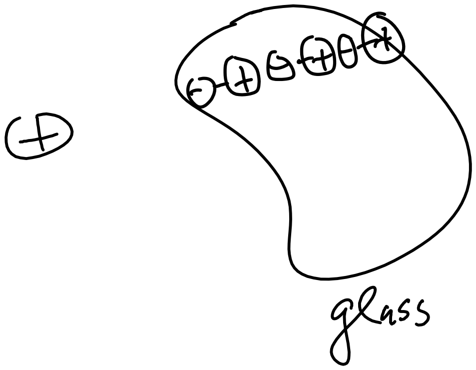
permanent dipole



alignment via rotation and energy dissapation



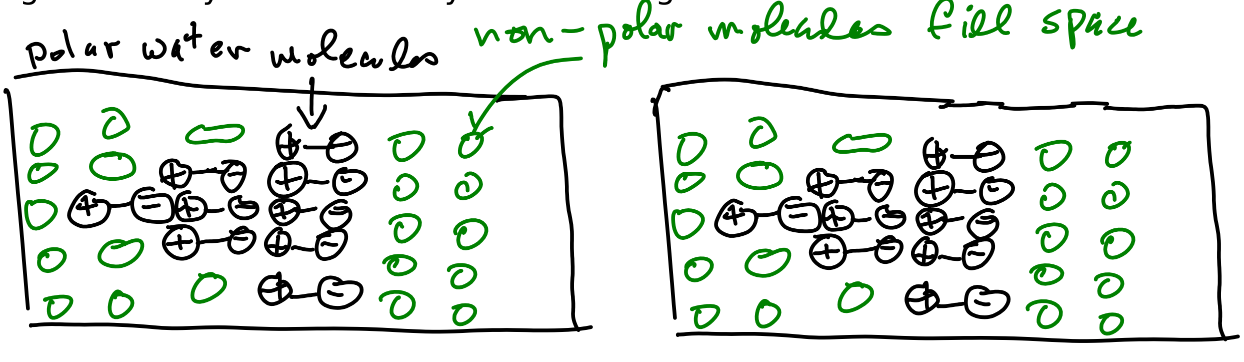
causal/creative: What happens to the atoms in the glass when a field is applied?

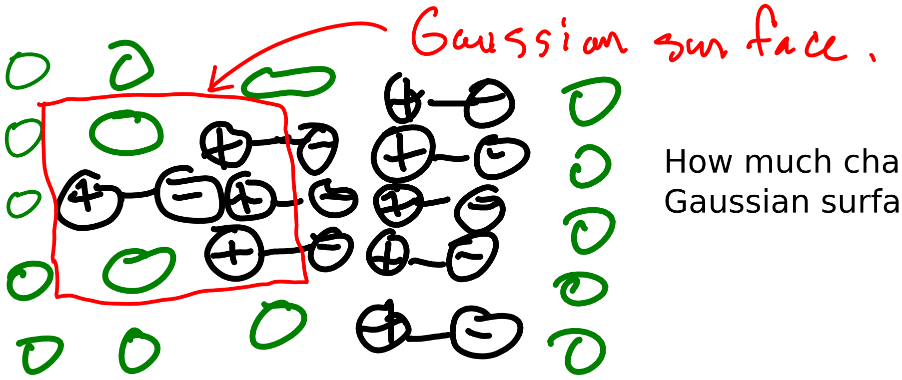


We will solve this problem by noting that the interior charges may cancel leaving only surface charges. To find E get rid of the glass atoms and find the total E due to the surface charge and initial charge.

Questions:

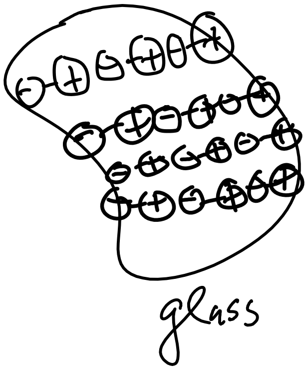
incongruous: Why isn't there any bound charge inside? That makes no sense!





How much charge is enclosed by this Gaussian surface?

congruous: I understand the model but how to I calculate the electric field?



(1) Assume that permanent dipoles are glued in place. Ex: water crystal or Barium Titanate which is piezoelectric.

(2) Assume that only we need only calculate the dipole field because the dipoles are so small (point dipoles not physical dipoles)

congruous: OK I understand the model but should I try to find E or V everywhere?

informational: OK I understand the model and that I need to calculate V
but what answer should I expect?

$$V = \frac{1}{4\pi\epsilon_0}\int ?$$

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho\, d\tau'}{r}$$

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma\, da'}{r} + \frac{1}{4\pi\epsilon_0}\int \frac{\rho\, d\tau'}{r}$$

↑ V due to surface charge (bound & not free)

↖ V from bound vol charge

$$V_1 = \frac{\vec{P}_1 \cdot \hat{r}_1}{4\pi\epsilon_0 r_1^2}$$

$$d\vec{p} = \vec{P}\, d\tau$$

↖ dipole mom

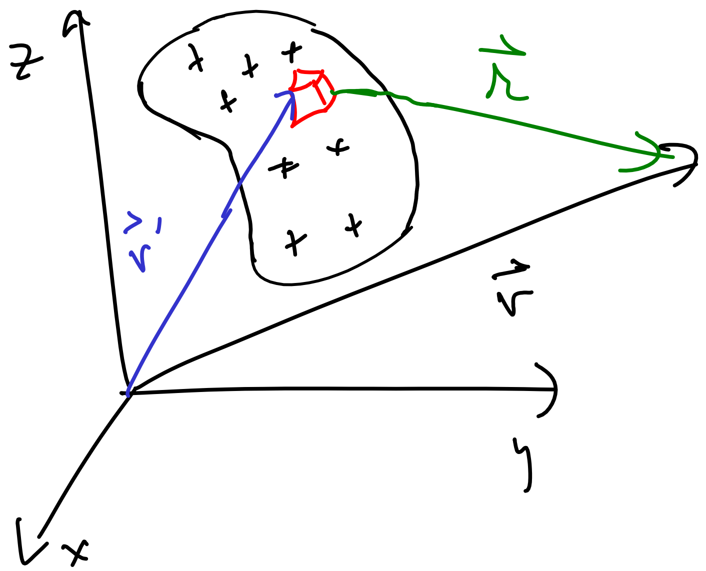
$$V_{tot} = \sum_i{}' V_i = \sum{}' \frac{\vec{P}_i \cdot \vec{r}_i}{4\pi\epsilon_0 r_i^2}$$

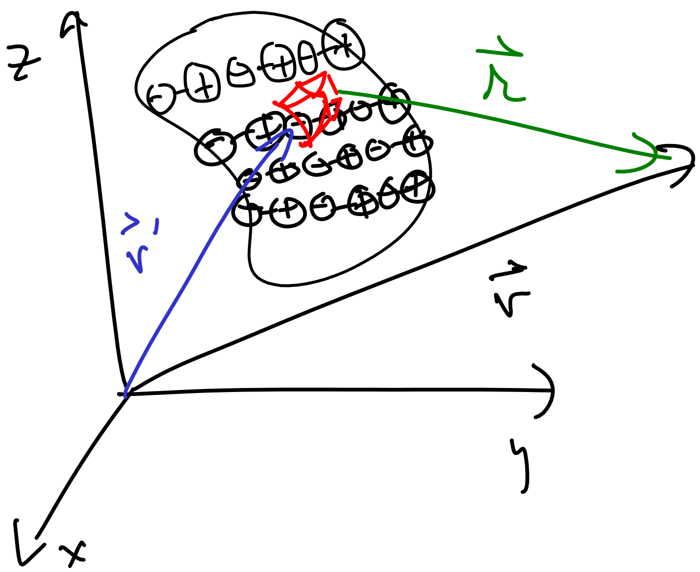
$$\sum_{\tau}{}' \rightarrow \int$$

congruous: I understand what we are doing but how do I calculate the integral?

analogy: How is this similar to going from sum to an integral in the voltage?

$$V_{\text{pt charges}} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho\, d\tau'}{\mathscr{r}} \qquad V_{\text{dipoles}} = \sum_{i=1}^{N} \frac{\vec{p}_i \cdot \vec{r}_i}{4\pi\epsilon_0 r_i^2}$$





$$d\vec{p} = \vec{P}\, d\tau$$

$$\uparrow \frac{\text{dipole moment}}{\text{vol}}$$

$$V_{\text{dipoles}} = \sum_{i=1}^{N} \frac{\vec{p}_i \cdot \hat{r}}{4\pi\epsilon_0 r_i^2}$$

$$\swarrow$$

$$dV = \frac{1}{4\pi\epsilon_0}\frac{\vec{P}\cdot(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\, dx'dy'dz'$$

Questions:

congruous: I understand where to start from and where to go but how do I get to an expression for the voltage in terms of "bound" charge instead of the dipole moment per unit volume?

$$V = \int dV = \frac{1}{4\pi\epsilon_0}\int \frac{\vec{P}\cdot(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\,dx'dy'dz'$$

$$V = \frac{1}{4\pi\epsilon_0}\int\underset{\uparrow}{\frac{\sigma\, da'}{r}} + \frac{1}{4\pi\epsilon_0}\int\frac{\rho\, d\tau'}{r}$$

Surface integral
maybe it came from application
of the divergence theorem.

If so then we need $\vec{\nabla}$ operator in integrand.

Homework problem 4. Show that

$$\vec{\nabla}'\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right) = \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}$$

$$V = \int dV = \frac{1}{4\pi\epsilon_0}\int \vec{P}\cdot\vec{\nabla}'\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right)\,dx'dy'dz'$$

We need to move the operator over to P (analogous to integration by parts).

Homework problem 5. Using the vector identity

$$\vec{\nabla}\cdot(f\vec{A}) = f\vec{\nabla}\cdot\vec{A} + \vec{A}\cdot\vec{\nabla} f$$

and the divergence theorem show

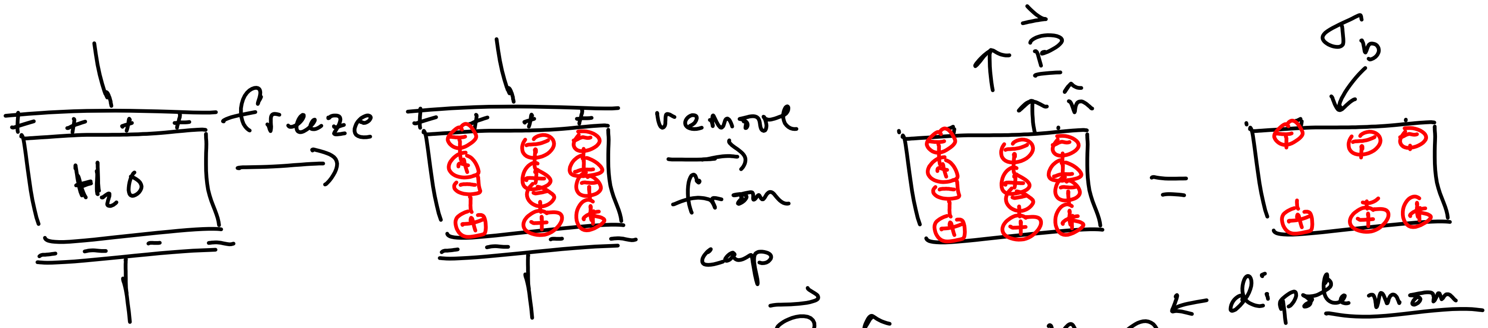
$$V = \frac{1}{4\pi\epsilon_0}\oint\frac{\vec{P}\cdot d\vec{a}'}{|\vec{r}-\vec{r}\,'|} + \frac{1}{4\pi\epsilon_0}\int\frac{-\vec{\nabla}'\cdot\vec{P}}{|\vec{r}-\vec{r}\,'|}\,dx'dy'dz'$$

Identify the bound surface and volume charges using our expected voltage.

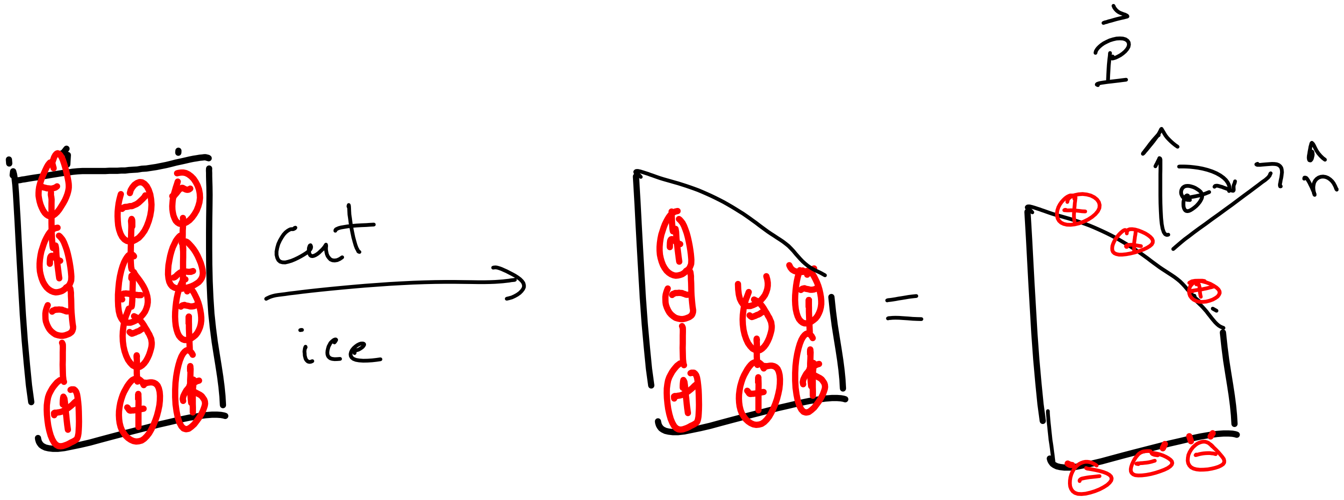
$$V = \frac{1}{4\pi\epsilon_0}\int\frac{\sigma_b\,da'}{\mathscr{r}} + \frac{1}{4\pi\epsilon_0}\int\frac{\rho_b\,d\tau'}{\mathscr{r}}$$

$$\sigma_b = \vec{P}\cdot\hat{n} \qquad\qquad \rho_b = -\vec{\nabla}'\cdot\vec{P}$$

Example: Put water in a charge capacitor. Freeze in the permanent dipoles.



$$\sigma_b = \vec{P}\cdot\hat{n} = \underset{\uparrow\ \text{molecules/vol}}{n}\ p \leftarrow \frac{\text{dipole mom}}{\text{molecule}}$$



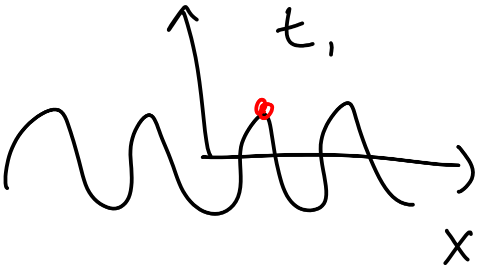
$$\sigma_b = \vec{P}\cdot\hat{n} = \sigma_b^{\perp}\cos\theta = np\cos\theta$$

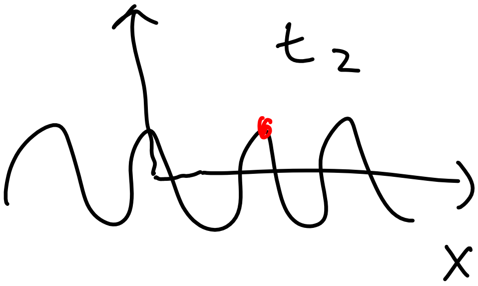
You have the same charge over a larger area so the charge density decreases.

Questions:

congruous: Now that we have the charge distribution how do I calculate E both inside and outside the cut slab of ice?

Retarded time method:



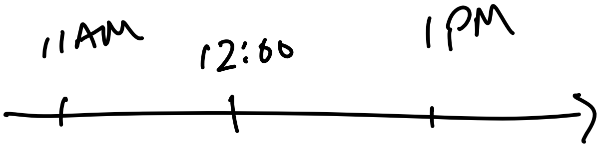


$$E(x,t) = E_0 e^{i\overbrace{(kx-\omega t)}^{\phi(x,t)\ \text{phase}}}$$

$$E(x=0,t) = E_0 e^{-i\omega t}$$

At some distance x the wave height is the same as it was at the origin at an earlier time.

$$t_{\text{retarded}} = t - \frac{x}{c}$$

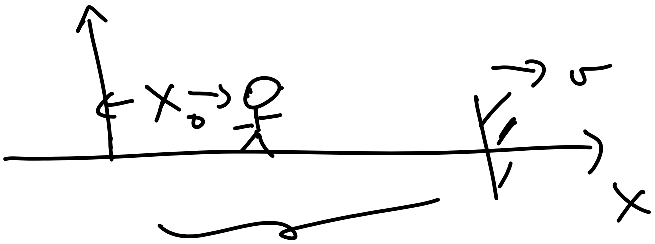


$t_{\text{now}} = t$

where is the future & past on this axis?

$$E(x,t) = E_0 e^{-i\omega t_{\text{retarded}}} = E_0 e^{-i\omega\left(t - \frac{x}{c}\right)}$$

Homework problem 6.) Derive the expression for the wave reflected from a mirror moving at constant speed using the retarded time method.



$$\phi(x=0,t) = -\omega t$$

$$\phi(x_0,t) = ?$$

$$x_{\text{mirror}} = L + vt$$

Look up retarded potential on wikipedia

Use the following search words on wikipedia
caltech.edu
moving charge
click on the first link to view the applet (make sure your computer can run java)

Homework problem 7.) Solve for the output voltage for this amplifier using a perturbative technique. Show that it results in the result from your electronics text.