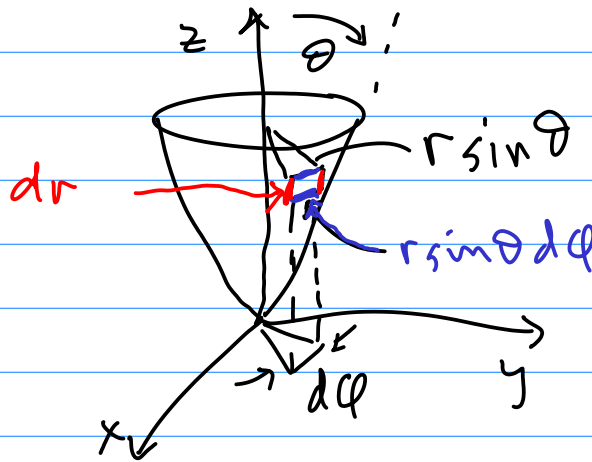


# Lecture 15 Shadowitz: 4-2 capacitance, 4-3 electrostatic energy

Add 5 points to all exams for the last part of the last problem.

InkSurvey:

$$d\vec{a} = r \sin\theta dr d\phi \hat{\theta}$$



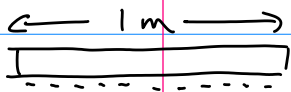
$$\text{Area} = \int_0^{\frac{2\pi}{s}} \int_0^s r \sin\theta dr d\phi = \frac{s^2}{2} 2\pi \sin\theta$$

What's the flux of E for a point charge at the origin through this cone?

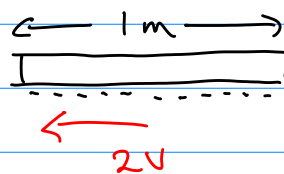
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{(x^2 + y^2 + z^2)^{3/2}} (x\hat{x} + y\hat{y} + z\hat{z}) = \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} \hat{r}$$

$$\vec{E} \cdot d\vec{a} = \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} \hat{r} \cdot r \sin\theta dr d\phi \hat{\theta}$$

InkSurvey:



$$\lambda_0 = \frac{1000 \text{ g}}{1 \text{ m}}$$



$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

Muddiest point question: how does velocity affect Q (causal/creative)?

What does charge invariance mean (informational)?

How do we know this (informational)?

ANS: Q is the same in all frames. Invariance means it doesn't change as viewed from different reference frames. If the electron in an atom, which moves at a much different speed from the proton had its charge dependent on speed then the light emitted from atoms would be much more difficult to model.

$$\lambda = \frac{1000q}{L_0 \sqrt{1 - \left(\frac{2v}{c}\right)^2}} \approx \frac{1000q}{L_0} \left( 1 + \frac{1}{2} \left(\frac{2v}{c}\right)^2 + \dots \right)$$

$\uparrow$   
 $\lambda_0$

The moving ruler has a higher charge density than a stationary charged ruler.

Muddiest points:

-Why do I ask for questions and why not ask for specific types of questions?

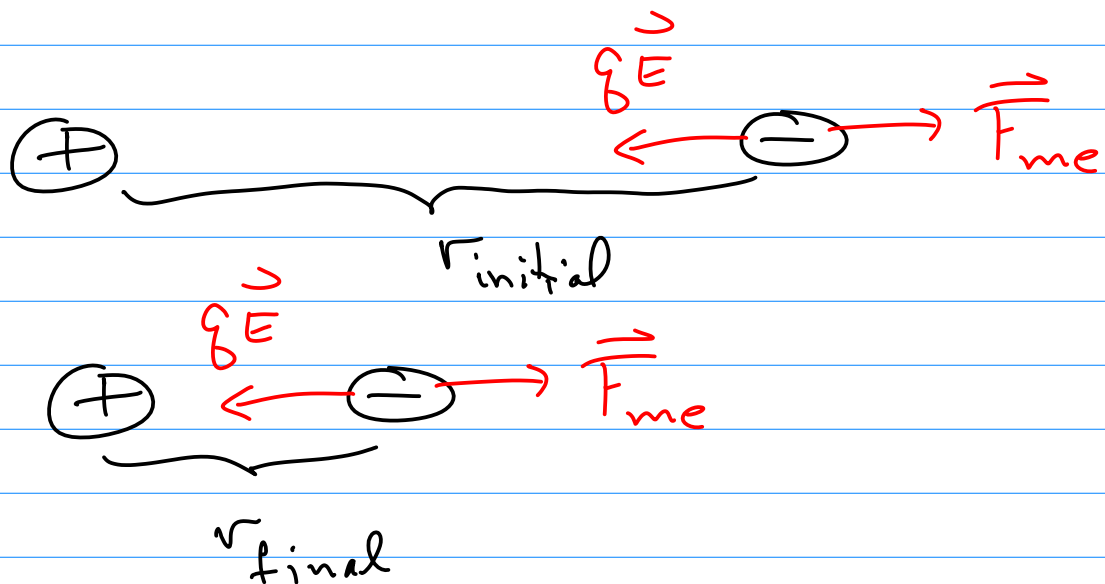
Normally I give you answers before you have asked any questions. Why would you care about such an answer?

-How are B and E related?

We will soon get to Faraday's law and the displacement current which relates these.

-What does negative work done by me mean?

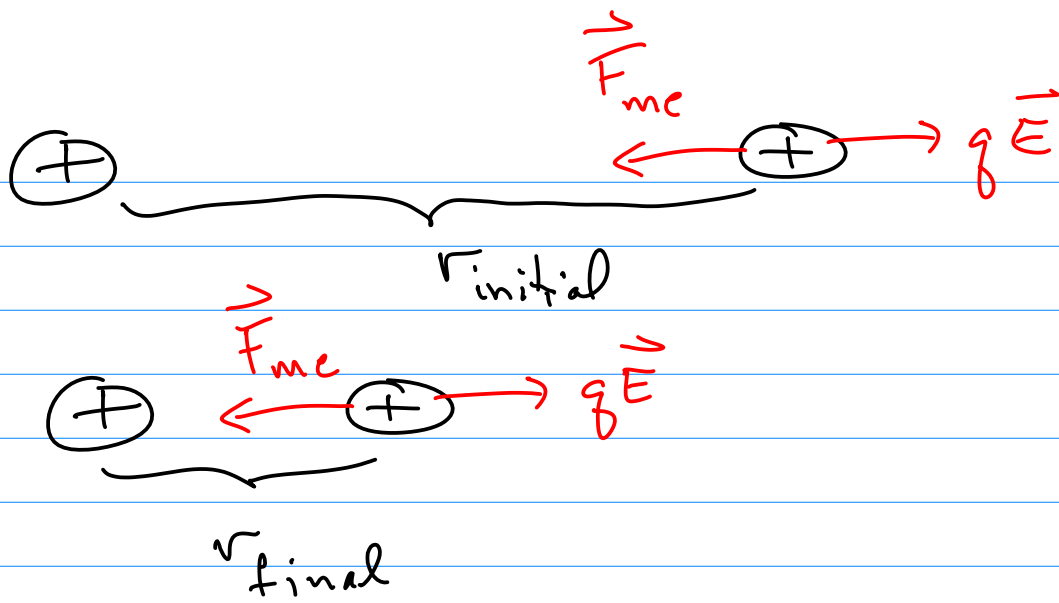
It says that you have to exert a force to the charges to keep them from coming together.



$$W_{me} = \int_{r_{int}}^{r_{final}} \vec{F}_{me} \cdot d\vec{r} \quad \text{Assume } |\vec{F}_{me}| = \text{constant}$$

$$F_{me} (r_{final} - r_{initial})$$

$W_{me}$  is neg if  $r_{final} < r_{initial}$



If the force I exert is in the direction of motion the work I do is positive. I have to push the charges together and I can get that work out when they are released.

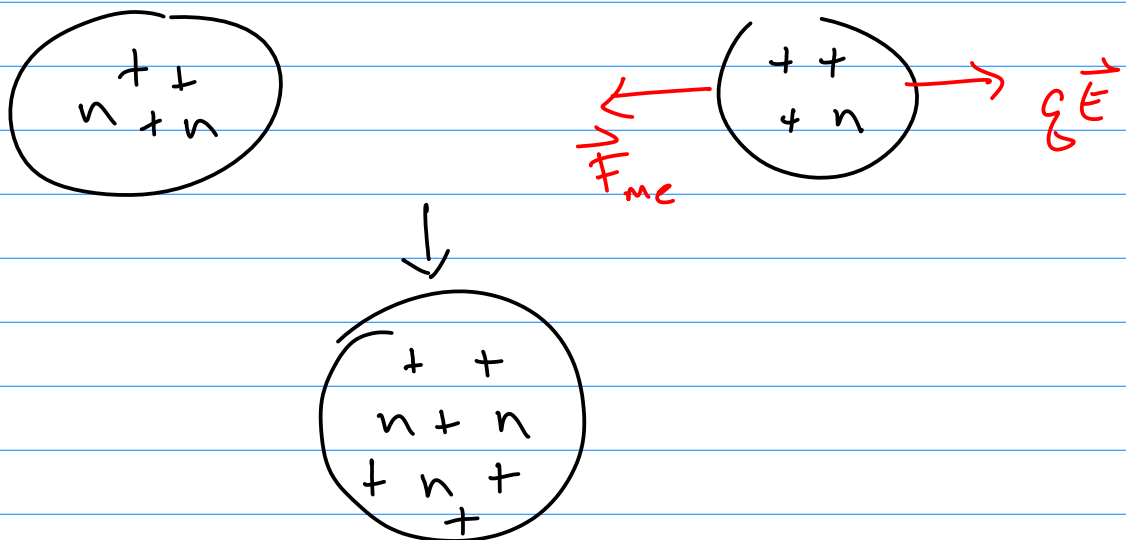
$$W_{me} = \Delta PE = PE_f - PE_i > 0 \Rightarrow PE_f > PE_i$$

-you put NaCl and take it apart and get zero net work. So what?

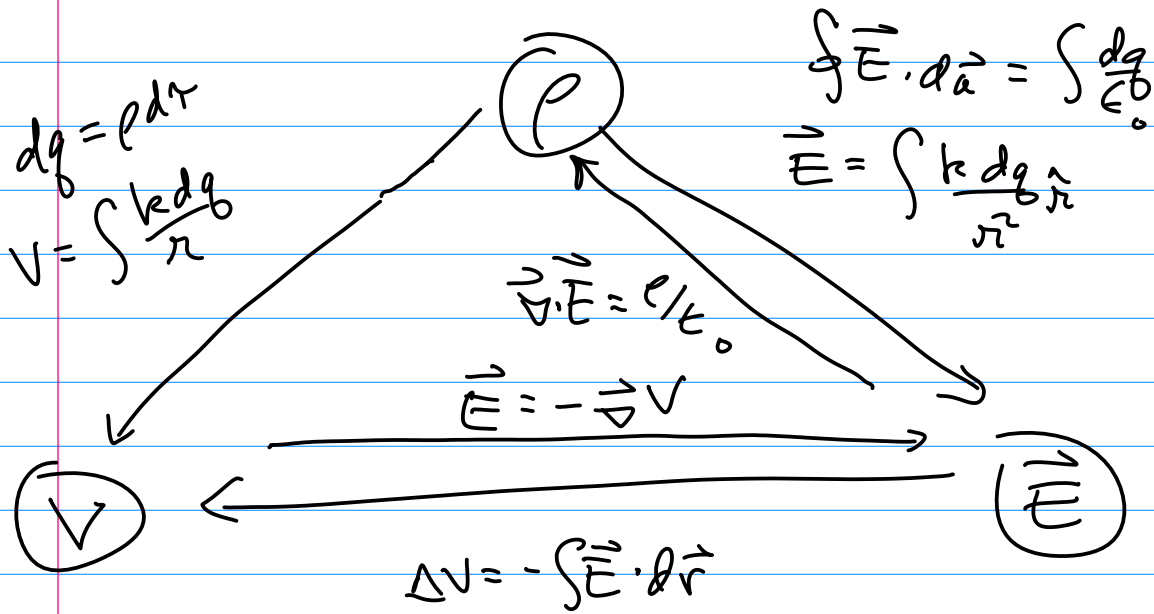
If you want to make a new material the first question to answer is its stability energetically.

-Why do charges stay together if it requires energy?

Here is an example in nuclear physics. Two nuclei are brought close enough so the nuclear force binds them into a larger nucleus. When they fission (a neutron splits them) they release their electrostatic potential energy. This is where the energy for a nuclear reactor comes from.



- Where are we going?



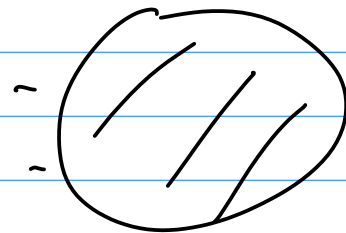
$\vec{F} = q \vec{E}$   
 $W_{me} = \int V dq = \Delta PE$

$W_{nc} = \Delta (KE + PE)$

Questions?

Muddiest point: Explain more on  $V dq$  build up of a charge distribution.

two conductors



$\Delta V \propto Q$

$\Delta V = \frac{Q}{C}$

$C = \frac{Q}{\Delta V}$

C is the capacitance

To bring in the first dq requires no work since V present is zero.

$$W_{me} = \int V dq = \int_0^{Q_f} \frac{q}{C} dq = \frac{1}{C} \frac{1}{2} Q_f^2$$
$$= \frac{1}{2} \frac{1}{C} (C \Delta V)^2 = \frac{1}{2} C \Delta V^2$$

Example: charging a capacitor and discharging it through a watermelon.

<http://www.youtube.com/watch?v=gj1pkyCL75E>

Questions:

incongruous: How did they fake this? Did they put dynamite in the melloon?

congruous: How do I calculate the effect knowing the energy stored in the caps?

modifying: Would the same thing happen to a balloon of water?

generalizing/analogy: How similar is this process to that using dynamite?

How would this differ if a high energy laser hit the watermelon?

causal/creative: Is the reason for the explosion vaporizing water due to the transfer of electrical energy into heat energy?

informational: How much energy was stored in the caps before discharge?

How can I estimate the KE of the bits after the explosion?

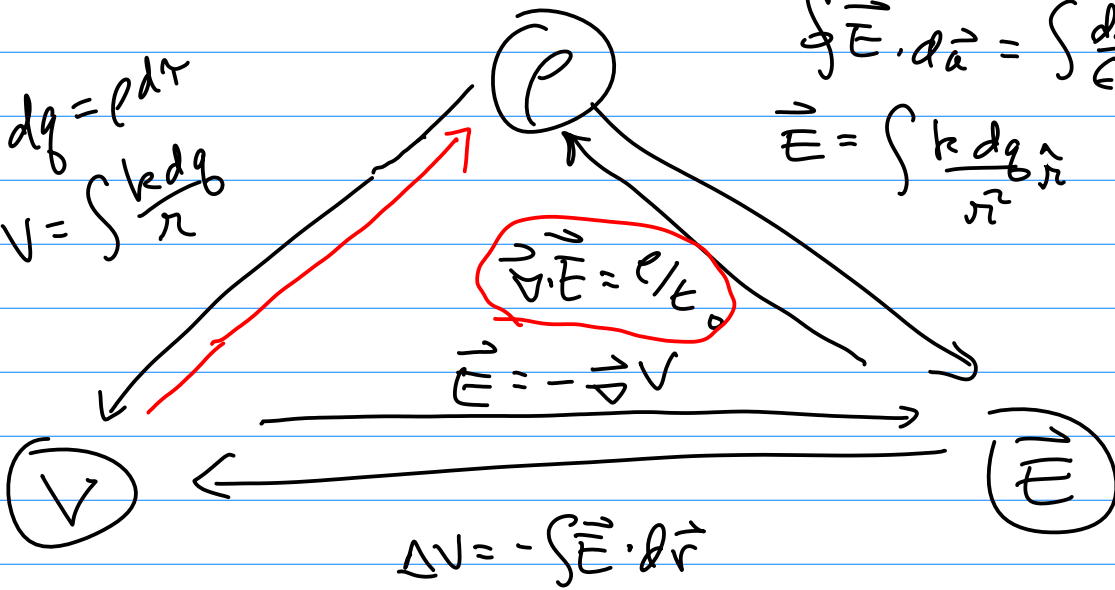
How far did the bits fly away?

$$dq = \rho dr$$

$$V = \int \frac{k dq}{r}$$

$$\oint \vec{E} \cdot d\vec{a} = \int \frac{dq}{\epsilon_0}$$

$$\vec{E} = \int \frac{k dq}{r^2} \hat{r}$$



$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$$

$$\vec{\nabla} \cdot (-\vec{\nabla} V) = -\nabla^2 V$$

$$\left. \begin{array}{l} \vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0 \\ \vec{\nabla} \cdot (-\vec{\nabla} V) = -\nabla^2 V \end{array} \right\} \nabla^2 V = -\rho/\epsilon_0$$

Given  $V$  find  $\rho$

$$\left( \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \right) \cdot \left( \hat{x} \frac{\partial V}{\partial x} + \hat{y} \frac{\partial V}{\partial y} + \hat{z} \frac{\partial V}{\partial z} \right)$$

$$= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

In free space  $\rho = 0$   $\nabla^2 V = 0$

1-D

$$\frac{d^2 V}{dx^2} = 0$$

