Phys 361 Homework 12 – Due Monday April 29th

Note: The first two problems are about material from earlier chapters and are meant to provide some structured review. They are *not* hints about the final – I haven't written that yet – so don't assume that this is the only stuff you need to know about.

1) Pollack and Stump 3.44. Nothing fancy here; just a little practice on basic integration and expansions.

2) Here's one that reminds me of perturbation theory in quantum mechanics. If you haven't seen that yet, this is the gist of it: You have a system, let's call it S_0, governed by the Schroedinger equation, and you'd like to solve for the allowed wavefunctions for S_0, but you can't. Too hard. But you *can* solve for the allowed wavefunctions for a *similar* system $S_{0'}$, so you do that and then add a small correction to make the solutions look like the solutions to S_0.

Let's take a sphere of dielectric material with susceptibility χ_e. Apply a uniform electric field E_0. The sphere will polarize, and the polarization will make an additional field, E_1 (see our notes or chapter 6.3 for the E-field made by a polarized sphere). If χ_e is small (and it often is), E_1 will be much smaller than E_0, so E_0 is approximately the field inside the sphere. But $E_0 + E_1$ is a *better* approximation to the field inside the sphere. We can refer to E_1 as a first-order correction.

Go through and find the polarization P_1 made from the field E_0, and the field E_1 made by P_1. Then find the polarization P_2 made by E_1, and the field E_2 made by P_2, and on and on until you see the pattern. You're going to end up with an infinite series of progressively smaller corrections. Sum the series and see what you get. How does the answer compare to the solution obtained by other means in chapter 6.5.2?

3) There are four different equations that are sometimes referred to as Faraday's law. They are:

(I) $\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d}{dt} \int \vec{B} \cdot \overrightarrow{dA}$ (II) $EMF = -\frac{d\Phi}{dt}$

(III) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (IV) $\oint \vec{E} \cdot \overrightarrow{dl} = -\int \frac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{dA}$

These are not all the same. Nor are they even all always true (we discussed this issue at the end of 361).

a) For each of the four equations, describe what it says in English, and what (if anything) makes it unique. Be sure to mention if there are any conditions that must be satisfied for the equation to be true.

b) Pick three pairs of the equations and discuss how the equations in each pair are similar, and how they differ. So, for example, discuss how equations (II) and (III) are the same and how they are not.

4) Maxwell's analysis of the integral form of Ampere's law rested in part on the realization that the area bounded by a closed loop is not uniquely defined. That is, there's more than one open surface bounded by any closed loop (infinitely many such surfaces, in fact).   But what often goes unmentioned is that the same issue arises in the integral form of Faraday's law. As you can probably guess, this doesn't end up being a big deal (otherwise we'd have a Faraday-Maxwell equation).

a) Come up with a concrete example to show that Faraday's law works regardless of choice of area associated with a loop. That is, specify a physically-allowable B-field, a closed loop, and two different areas bounded by that loop and show explicitly that the flux through the two different areas is the same (you don't have to show explicitly that if it's a time varying field the rate of change of the flux is the same in either case; that follows rather directly).

b) Now show that if you have a B field *with divergence*, things break down. That is, you can find two different areas bounded by one loop such that the two areas have different fluxes passing through them. That's bad.

c) It would seem that if nature allowed for B fields with divergence, Faraday's law would have to be changed. Having seen the mathematical derivation of the Maxwell correction to Ampere's law, you should now be able to derive an appropriate correction to Faraday's law that allows for B-fields with divergence. Start by modifying the $\nabla \cdot \vec{B}$ equation and by writing a new continuity equation pertaining to magnetic monopoles.