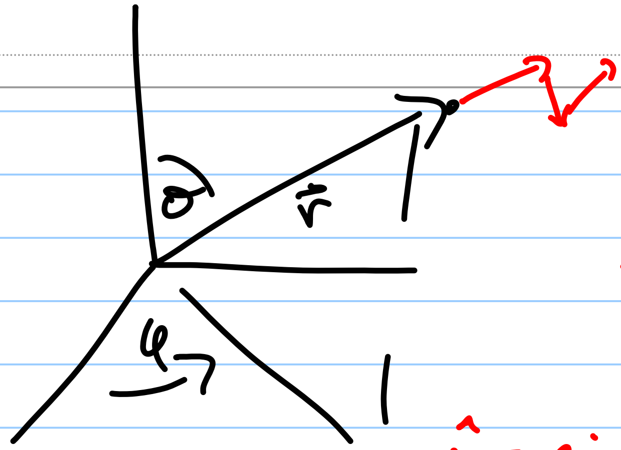

$$\vec{r} = r\hat{r} + \theta\hat{\theta} + \varphi\hat{\varphi}$$

$\theta = 45^\circ$, $\varphi = 45^\circ$

$$\hat{r} = \sin\theta\cos\varphi\,\hat{x} + \sin\theta\sin\varphi\,\hat{y} + \cos\theta\,\hat{z}$$

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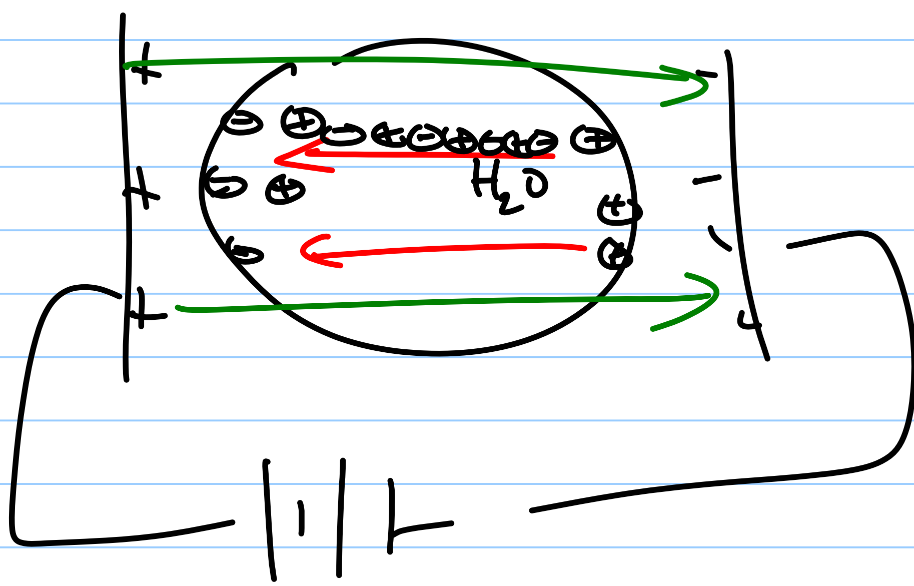
understand electric fields in materials



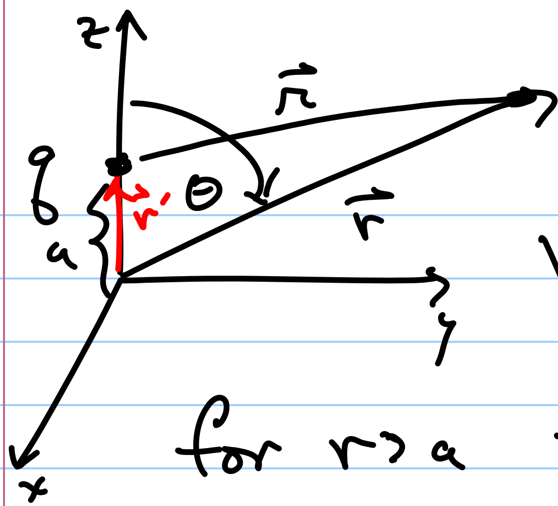
two types of problem

- summation

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')\,d\tau'}{r}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho\,d\tau'}{r^2}\hat{r}$$

- bndry value given V on metal find V & E everywhere



find $V(\vec{r})$

law of cosines

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{\mathscr{r}} = \frac{q}{4\pi\epsilon_0}\frac{1}{(r^2+a^2-2ar\cos\theta)^{1/2}}$$

for $r > a$ far away: factor r^2 out of

$$\left[r^2\left(1+\underbrace{\frac{a^2}{r^2}-\frac{2ar\cos\theta}{r^2}}_{\epsilon \text{ small}}\right)\right]^{1/2}$$

$$\frac{1}{(\ )^{1/2}} = \frac{1}{r(1+\epsilon)^{1/2}} = \frac{1}{r}\left[1-\frac{1}{2}\left(\frac{a^2}{r^2}-\frac{2a}{r}\cos\theta\right)+\frac{3}{8}\left(\frac{a^2}{r^2}-\frac{2a}{r}\cos\theta\right)^2+\cdots\right.$$

taylor expn

arrange in powers of $\frac{a}{r} \ll 1$

$$\frac{1}{r}\frac{1}{(1+\epsilon)^{1/2}} = \frac{1}{r}\left[1+\left(\frac{a}{r}\right)\underbrace{\cos\theta}_{P_1}+\left(\frac{a}{r}\right)^2\underbrace{\left[\frac{3\cos^2\theta-1}{2}\right]}_{P_2}+\cdots\right)$$

$$V = \frac{q}{4\pi\epsilon_0}\frac{1}{r}\sum_{l=0}^{\infty}\left(\frac{a}{r}\right)^l P_l(\cos\theta) \qquad r > a$$

one pt charge

charge distribution integrate $q \to \rho\, d\tau$

$$V = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\vec{r}')\,d\tau'}{\mathscr{r}}$$

on z axis

$\rho(r') = \lambda(z')\,\delta(x')\,\delta(y')$

$$V = \frac{1}{4\pi\epsilon_0}\iiint \delta(x')\,dx'\,\delta(y')\,dy'\,\frac{\lambda(z')\,dz'}{(r^2+z'^2-2z'r\cos\theta)^{1/2}}$$

$$= \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(z')}{r}\sum_{l=0}^{\infty}\left(\frac{z'}{r}\right)^{l} P_l(\cos\theta)\,dz'$$

$$= \frac{1}{4\pi\epsilon_0}\sum_l \frac{P_l(\cos\theta)}{r^{l+1}}\underbrace{\int \lambda(z')(z')^l\,dz'}_{M_l}$$

M_l axial multipole moment

find M_0

$$\int \underbrace{\lambda(z')}_{\text{charge/length}}\ \underbrace{z'^0}_{1}\ \underbrace{dz'}_{dz'} = Q_{tot} \qquad M_0$$

$V \propto \frac{1}{r}$ monopole

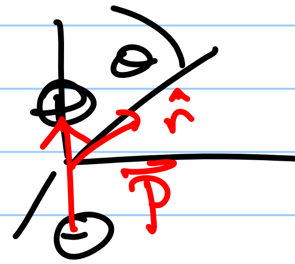
$V \propto \frac{1}{r^2}$ dipole

$\vec{p}\cdot\hat{r}$

dipole

$$V_1 = \frac{1}{4\pi\epsilon_0}\,\frac{M_1\cos\theta}{r^2}$$

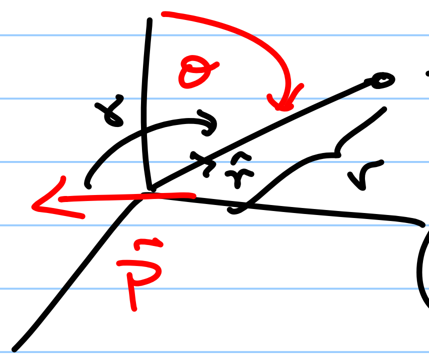
coord indep



M_1 mag dipole moment

direction $|\vec{p}|$

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

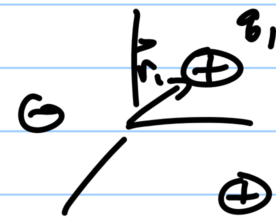


$\vec{p}\cdot\hat{r} = p\,1\cos\gamma$

$\gamma = \theta + 90$

$\vec{p}\cdot\hat{r} = p\cos(90+\theta)$

axial moments



general charge dist.

$$\boxed{\vec{p} = \sum_{i=1}^{N} q_i\,\vec{r}_i} \to \int$$