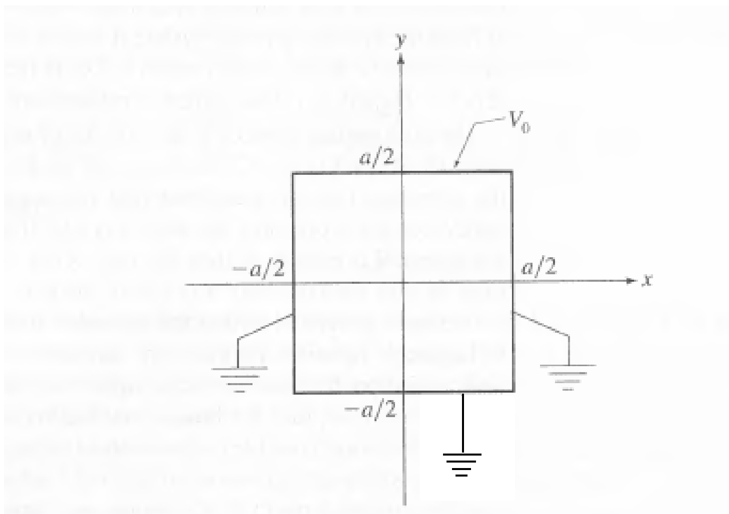
Phys 462 Homework 2

1) (from Pollack and Stump 5.7)



We have a hollow conducting pipe with a square profile, as shown. The sides at x = +a/2, x = -a/2, and y = -a/2 are all grounded. The side at y = +a/2 is held at some fixed potential V_0. Show that the potential everywhere inside the pipe is given by:

$$V(x,y) = \frac{4V_0}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\cos\left[(2n+1)\frac{\pi x}{a}\right]\frac{\sinh\left[(2n+1)\pi\left(\frac{y}{a}+\frac{1}{2}\right)\right]}{\sinh[(2n+1)\pi]}$$

You might need to use trig identities for hyperbolic functions to make the answer that comes out look like the one shown (I had to use one of the addition identities and a double angle formula).

Another hint: When you go to find your series coefficients by way of Fourier's trick, you might find that you need to use two conditions to find two coefficients. I personally found that exploiting the even/oddness of sinh and cosh made the coefficient solving easier. But if you get through the problem without ever noticing what I'm talking about, then don't worry about it.

2) Okay, so we know that current doesn't drive itself. In a current-carrying wire, there's an electric field pushing the conduction electrons along. But how does that field come about? In electrostatics fields come from charges. And there can't be any net charge inside the conductor. The battery might be really far away, leaving it an inappropriate source. So when we think real hard we get forced into the conclusion that a current-carrying wire has a *surface* charge distribution σ that is responsible for making the E-field that pushes the current. Most people find this rather surprising the first time they hear about it.

Consider a coax cable. There is a *solid* (3D) cylindrical wire of radius a carrying a uniform current I in the $\hat{k}$ direction. Surrounding that is a *hollow* (2D) cylindrical sheath of radius b carrying the return current back in the other direction.

Your ultimate goal is to find the surface charge density on the solid interior wire (at r = a). To do that you'll have to find the potential in various regions and then invoke one or more general boundary conditions to get the charge.

Some hints/suggestions:

You can't assume the potential is independent of z in this problem (why not?). But you can assume rotational invariance (potential <u>is</u> independent of φ). In other words, you'll need to find $V(r, z)$ in cylindrical coordinates.

Start from the very beginning ($\nabla^2 V = 0$), assume a product solution in cylindrical coordinates, and go from there.

The trickiest part of the problem is realizing that the separation constant has to be zero. This significantly constrains what the solution can be, and makes the problem solvable in a reasonable amount of time. Explain how we can conclude that that constant is zero.

You may be able to guess the form of the electric field that's driving the current, and confirm it (or infer it in the first place) from Ohm's law. That'll give you some leverage regarding the potential for r < a. Remember, any real conductor has at least a little resistance, so Ohm's law is relevant.

You'll also need the electric field outside the entire thing (for r > b). You might reasonably guess E = 0 based on a simple Gauss's law argument, but you might also reasonably second guess yourself since this problem doesn't have translational symmetry. But as it turns out, E really is zero out there. Use that for free if you need it (I needed it).

3) a) $\oint \vec{A} \cdot \overrightarrow{dl}$ may or may not be a quantity that you've worked with before, but it can be re-represented as something else that is very familiar. What is it, and how do you know?

b) If you've figured out (a), you can now write an equation of the form $\oint \vec{A} \cdot \overrightarrow{dl}$ = (blah), where blah is a scalar quantity. This equation is structurally identical to Ampere's law. Use this equation to figure out a vector potential describing an infinite solenoid with turn density n, current I, and radius a. Consider the vector potential both inside and *outside* the solenoid. Note: You do need to assume the expression for the solenoid's magnetic field in order to use this technique to get A.

c) Double check that your vector potential generates the correct B-field inside and outside the solenoid. Also check your vector potential to see if it's in any particular gauge.

d) Having a look back at what you've done, you should be able to convince yourself that A *can't* be zero outside the solenoid, even though B is zero out there. Is this bad?