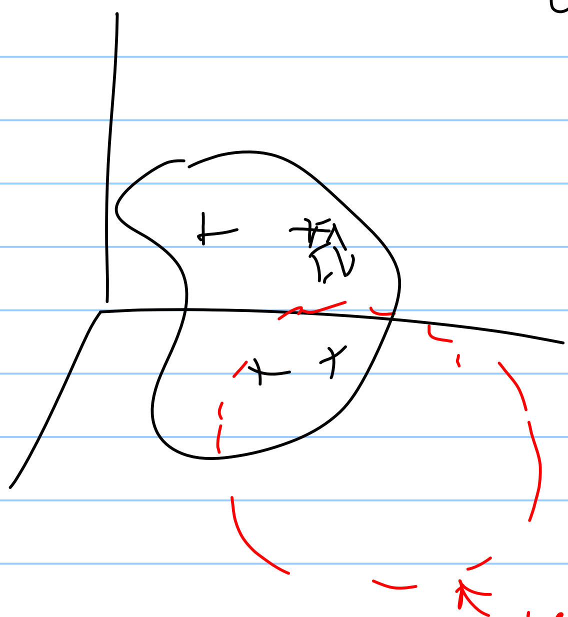
Note Title 1/21/2009

$$W = \frac{\epsilon_0}{2}\left[\oint V\vec{E}\cdot d\vec{a} + \underline{\underline{\int E^2 d\tau}}\right]$$

let surface $\to \infty$



doesn't enclose all the charge so invalid surface

$V \propto \frac{1}{r}$

$E \propto \frac{1}{r^2}$

$da \propto r^2$

$V E\, da \propto \frac{1}{r}$ $r \to \infty$

$\oint \to 0$

point charge the self energy included

$R \to 0$

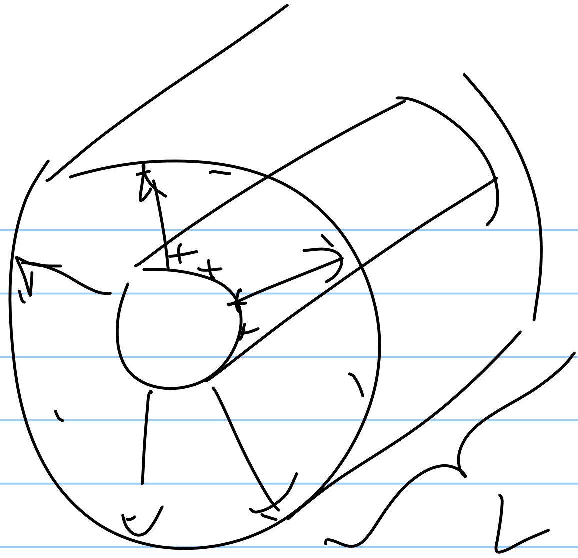
$$\cdot\ \text{Energy} = \frac{1}{2}\int_{\text{all space}} \epsilon_0 E^2\, \underline{\underline{d\tau}}_{\text{vol}} = \infty$$

$E \propto \frac{1}{R^2} \to \infty$

$R \to 0$

$\frac{1}{2}\epsilon_0 E^2$ energy density $\left(\frac{J}{m^3}\right)$

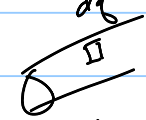
Ex:



Cap.

Fundamental Prin 1.) $\int dq\, V$ ↑ diff V

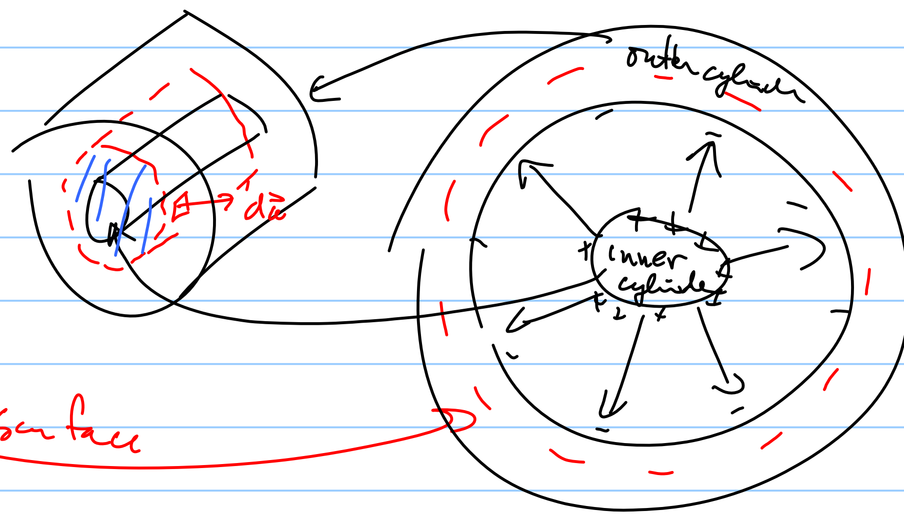
2.) $\frac{1}{2}\int dq\, V$



$V = -\int \vec{E}\cdot d\vec{l}$

↑ Gauss's Law

3.) $W_{me} = \frac{\epsilon_0}{2}\left[\oint V\vec{E}\cdot d\vec{a} + \int E^2 d\tau\right]$

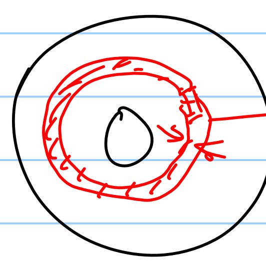


E = 0 on surface

end view

$\frac{1}{2}\epsilon_0 \int E^2 d\tau$

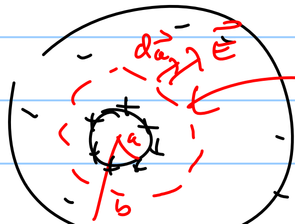
↑ volume element:



$2\pi r\, dr$

$L\int_0^R 2\pi r\, dr = \frac{2\pi R^2}{2}L = \pi R^2 L$

Use Gauss Law to find E



$\oint \vec{E}\cdot d\vec{a}$

$E\, 2\pi r L = \frac{Q_{enc}}{\epsilon_0} = \frac{\sigma 2\pi a L}{\epsilon_0}$

$$E = \frac{\sigma\, 2\pi a L}{2\pi r L \epsilon_0} = \frac{\sigma a}{\epsilon_0 r}$$

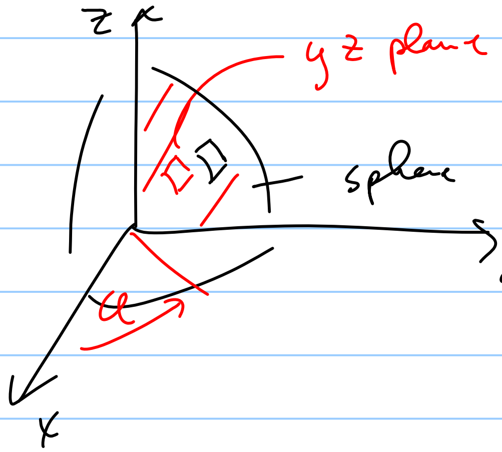
$$d\tau = L\, 2\pi r\, dr$$

$$\sigma = \frac{Q}{2\pi a L}$$

Divergence theorem:

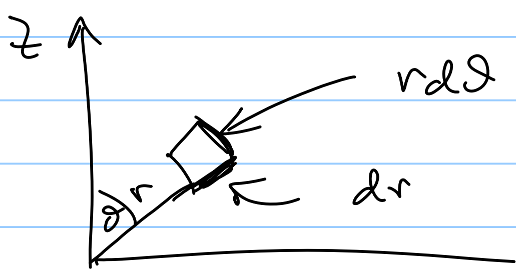
$$\oint \underset{\uparrow \atop \vec{v}}{\vec{E}} \cdot d\vec{a} = \int \nabla \cdot \underset{\uparrow \atop \vec{v}}{\vec{E}}\, d\tau$$

$$\vec{v} = r^2 \cos\theta\, \hat{r} + r^2 \cos\phi\, \hat{\theta} - r^2 \underline{\cos\theta \sin\phi\, \hat{\phi}}$$



$$\vec{E} = \int \frac{k\, dq}{r^2} \hat{r}$$

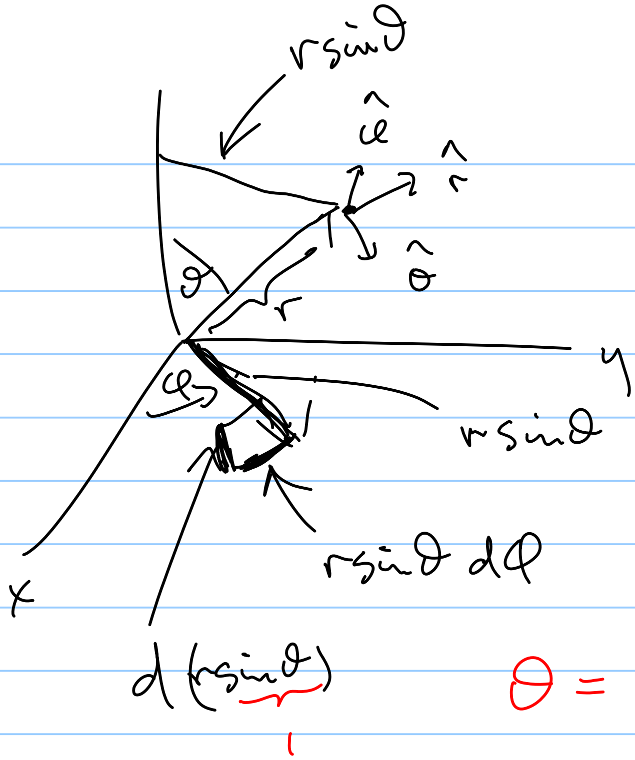
vector: want $\hat{x}, \hat{y}, \hat{z}$ so unit vectors don't depend on variable of integration



$$da = r\, dr\, d\theta$$

$$d\vec{a} = r\, dr\, d\theta\, \hat{\phi} \quad \text{at } \phi = \pi/2$$

$$\int_0^{\pi/2} \int_0^R \vec{v} \cdot d\vec{a} \Big|_{\phi = \pi/2}$$



$$da = r\underbrace{\sin\vartheta}_{1}\,d\varphi\,dr$$

$$\theta = \pi/2$$