PH320 exam 3: 4/18/08

Do any 4 problems. All are worth 25 points. No calculators. Closed book, although you may use 3 crib sheets (Letter or A4 size).

1. Let $\hat{H}$ be a Hamiltonian whose matrix representation is:

$$\begin{bmatrix} E_1 & \epsilon \\ \epsilon & E_2 \end{bmatrix}$$

   (a) Compute the energy levels.

   (b) Show that the presence of the off-diagonal term eliminates any possibility of degeneracy.

2. If you choose to do problem 2 you must use the following problem solving strategy:

   10 pts Write down the fundamental principles involved in the solution.

   10 pts Outline the method of solution.

   5 pts Check the limits of the solution

   An electron is confined to a 3D infinite potential cube on $[0, L] \times [0, L] \times [0, L]$.

   (a) What is the normalized ground state wavefunction?

   (b) Make a table of the first 10 energy levels labeled by their 3 quantum numbers. Show the degeneracy of each level.

3. (a) What does it mean for two observables to be *compatible.*

   (b) Prove that $Af = \alpha f$ and $Bf = \mu f$ implies that $[A, B] = 0$.

4. Let $\psi_1(x)$ be the ground state wavefunction for a particle in an infinite one-dimensional well on $[0, 1]$.

   (a) Compute $\phi_1(p)$.

   (b) Derive **but do not evaluate** and integral for the expectation $\langle\phi|p^2\phi\rangle$. Simplify the integral as much as possible.

5. Compute the most probable value of r for the ground state of Hydrogen.

6. (a) Explain the physical significance of an operator (associated with some physical observable) commuting with the Hamiltonian.

   (b) An electron decays from the first excited state to the ground state of Hydrogen. What are the wavelength and frequency of the emitted photon.

7. Suppose you have a population of identically prepared atoms in a particular excited state and observe their decay to the ground state. It is observed that the emitted photons will have a spread of energies peaked around the theoretical energy difference between the two states.

   (a) Why is this?

   (b) Is this decay of excited states consistent with what you know about stationary states? Explain.

Some potentially useful factoids

- h = 6.6 $\times 10^{-34}$ m^2 kg / s
- c = 3 $\times 10^8$ m/ s
- 1 ev = 1.6 $\times$ 10^{-19} J
- m_e = 9 $\times 10^{-31}$ Kg
- a_0 = .05 nm
- 1 eV = 6 $\times$ 10^{-19} J
- $\int \sin^2(x)dx = \frac{x}{2} - \frac{1}{4}\sin(x)$
- $\int_0^\infty re^{-r}dr = 1$