

Linear dielectrics

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

Gauss's law

$$\nabla \cdot \vec{E} = \frac{\rho_f}{\epsilon_0} = \frac{\rho_f}{\epsilon_0} + \frac{\rho_b''}{\epsilon_0} - \nabla \cdot \vec{P} = \rho$$

$$\epsilon_0 \nabla \cdot \vec{E} = \rho_f - \nabla \cdot \vec{P}$$

$$\nabla \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho_f$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 \vec{E} + \epsilon_0 \chi_e \vec{E} = \underbrace{\epsilon_0 (1 + \chi_e)}_{\epsilon} \vec{E} = \epsilon \vec{E}$$

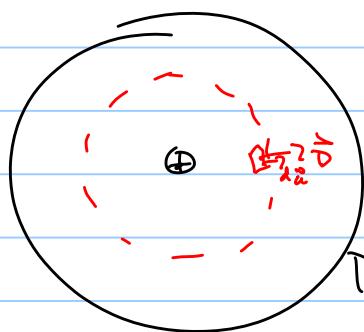


$$D = \sigma_f = \epsilon E$$

$$E = \frac{\sigma_f}{\epsilon} \quad P =$$

$$1.) \text{ find } D \text{ in terms of } \sigma_f \text{ or } \rho_f \quad 2.) \quad D = \epsilon E \Rightarrow E = \frac{D}{\epsilon}$$

$$3.) \quad P = \epsilon_0 \chi_e E \quad 4.) \quad \sigma_b = \vec{P} \cdot \hat{n} \quad \rho_b = -\nabla \cdot \vec{P}$$



$$\nabla \cdot \vec{D} = \rho_f \quad \oint \vec{D} \cdot d\vec{a} = Q_f$$

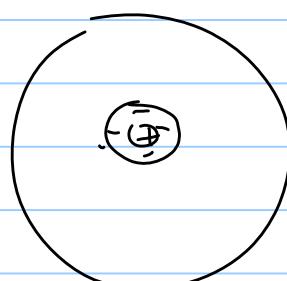
$$\oint |\vec{D}| (d\vec{a}) \cos \phi = D \oint d\vec{a}$$

$$D 4\pi r^2 = Q_f$$

$$\vec{D} = \frac{Q_f}{4\pi r^2} \hat{r} \quad \vec{E} = \frac{Q_f}{4\pi \epsilon r^2} \hat{r}$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E} = \frac{\epsilon_0 \chi_e Q_f}{4\pi \epsilon_0 (1 + \chi_e)} \frac{\hat{r}}{r^2}$$

$$\rho_b = -\nabla \cdot \vec{P} = -\frac{\chi_e Q_f}{4\pi (1 + \chi_e)} \frac{1}{r^2} \frac{1}{\delta^3(r)}$$

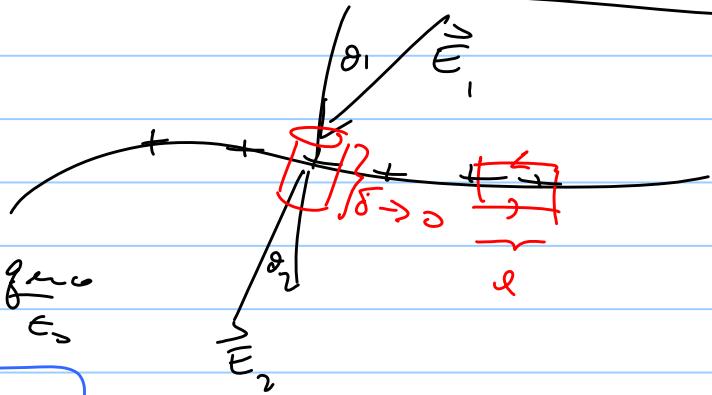


$$\nabla_b = \vec{P} \cdot \hat{n} = \vec{r}$$

boundary conditions:

- No dielectric

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad \oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$



$$E_{\text{above}}^{\perp} - E_{\text{below}}^{\perp} = \frac{\sigma}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = \sigma \quad \oint \vec{\nabla} \times \vec{E} \cdot d\vec{a} = \oint \vec{E} \cdot d\vec{l} = 0$$

$$E_{\text{above}}^{\parallel} \cdot \ell - E_{\text{below}}^{\parallel} \cdot \ell = \sigma$$

$$E_{\text{above}}^{\parallel} = E_{\text{below}}^{\parallel}$$

Given $\vec{E}_1 \perp \tau$ find \vec{E}_2

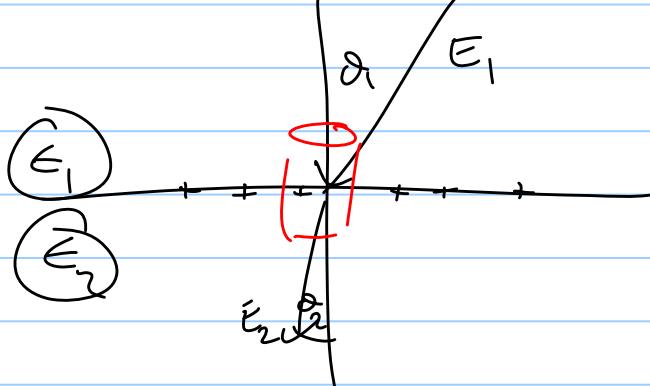
$$E_1 \cos \theta_1 - E_2 \cos \theta_2 = \sigma$$

$$E_1 \sin \theta_1 = E_2 \sin \theta_2$$

Dielectric

$$\vec{\nabla} \cdot \vec{D} = \rho_f \quad \text{or} \quad \oint \vec{D} \cdot d\vec{a} = Q_f$$

$$D_{\text{above}}^{\perp} - D_{\text{below}}^{\perp} = \sigma_f$$



$$\vec{\nabla} \times \vec{D} = \vec{\nabla} \times (\epsilon_0 \vec{E} + \vec{P}) = \epsilon_0 \vec{\nabla} \times \vec{E} + \vec{D} \times \vec{I} = \vec{\nabla} \times \vec{P}$$

$$\text{Stokes} \quad \oint \vec{D} \cdot d\vec{a} = \oint \vec{D} \cdot d\vec{l} = \oint (\epsilon_0 \vec{E} + \vec{P}) \cdot d\vec{l}$$

$$\epsilon_0 \oint \vec{E} \cdot d\vec{l} = 0$$

$$\oint \vec{D} \cdot d\vec{l} = \oint \vec{P} \cdot d\vec{l}$$

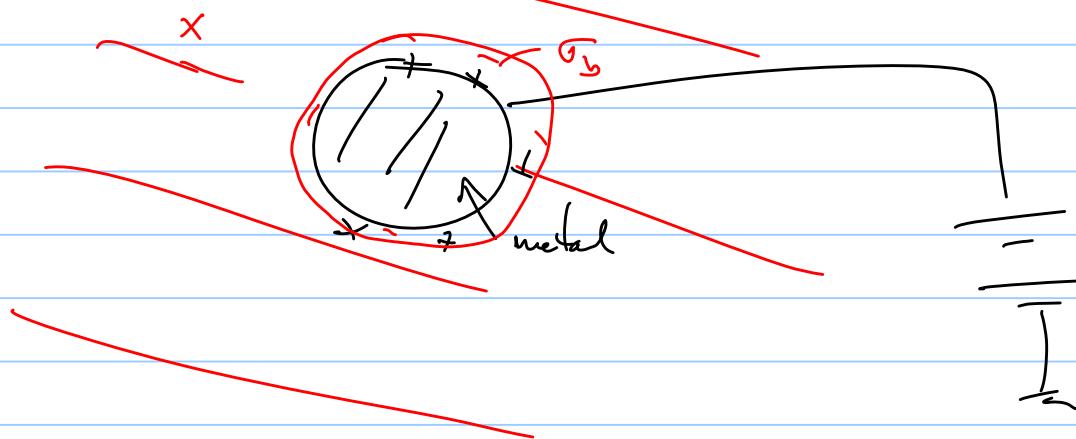
$$P_{\text{ext}}'' - D_{\text{below}}'' = P_{\text{ext}}'' - P_{\text{below}}$$

linear dielectrics

$$\vec{P} = \epsilon_0 \chi_e \vec{E} \quad \vec{D} = \epsilon \vec{E}$$

$$f_b = - \nabla \cdot \vec{D} = - \nabla \cdot \frac{\epsilon_0 \chi_e \vec{D}}{\epsilon} \rightarrow - \frac{\epsilon_0 \chi_e}{\epsilon} \nabla \cdot \vec{D}$$

$$f_b = - \epsilon_0 \frac{\chi_e}{\epsilon} f_f$$



$\nabla^2 V = 0$ Laplace's eqn holds in dielectrics