PHGN341 Thermal Physics Exam 2

(maximum = 60 points)

1.

Consider a Fermi gas of N spin 1/2 particles IN TWO DIMENSIONS, confined to a square of area L^2.

(a) Find the Fermi energy ε_F in terms of $n = N/L^2$. (7 points)

(b) Show that the density of states $\mathcal{D}(\varepsilon)$ is

$$\mathcal{D}(\varepsilon) = N/\varepsilon_F$$

for $\varepsilon \geq 0$, and 0 otherwise. (7 points)

(c) Show that

$$\mu = \tau \ln(\exp(\varepsilon_F/\tau) - 1)$$

Use

$$\int dx \frac{1}{e^x + 1} = -\ln(1 + e^{-x})$$

(6 points)

2.

Consider a Bose gas of N spin 0 particles IN TWO DIMENSIONS, confined to a square of area L^2.

(a) Show that the density of states is

$$\mathcal{D}(\varepsilon) = \frac{m}{\hbar^2}\frac{L^2}{2\pi}$$

for $\varepsilon \geq 0$, and 0 otherwise. (7 points)

(b) Define the two dimensional quantum density by

$$\rho_Q = \frac{m\tau}{2\pi\hbar^2}$$

and show that

$$\lambda = \exp(\mu/\tau) = 1 - \exp(-n/\rho_Q)$$

where $n = N/L^2$. Use

$$\int dx \frac{\lambda}{e^x - \lambda} = \ln(1 - \lambda e^{-x})$$

(7 points)

(c) What is the Bose-Einstein condensation temperature in this system? (6 points)

3.

Consider electron-positron annihilation in a gas of electrons and positrons and photons. We have the reaction

$$e^+ + e^- \leftrightarrow \gamma$$

Photons have no chemical potential, so $\mu_\gamma = 0$.

Electrons and positrons have the same mass, and so their energy is given by $\varepsilon_p = \sqrt{(mc^2)^2 + (pc)^2}$ where $p = \frac{\hbar c \pi}{L} n$ with $n = |\vec{n}|$, as usual.

We shall only consider the limit $\tau >> mc^2$ where we may take

$$\varepsilon_p = pc$$

The density of states is then

$$\mathcal{D}(\varepsilon) = \frac{L^3}{(\hbar c)^3 \pi^2} \varepsilon^2$$

(a) N_+ is the number of positrons and N_- is the number of electrons. Assuming that the system has no net charge, so that $N_+ = N_-$, show that this, combined with the condition for equilibrium in the reaction, implies $\mu_+ = \mu_- = 0$ in equilibrium. You do not need to do any integrals to prove this. (10 points)

(b) Using

$$\int_0^\infty dx \frac{x^2}{e^x + 1} = \frac{3}{2}\zeta(3)$$

($\zeta(3)$ is Riemann's zeta function) find the equilibrium density of electrons and positrons. (10 points)