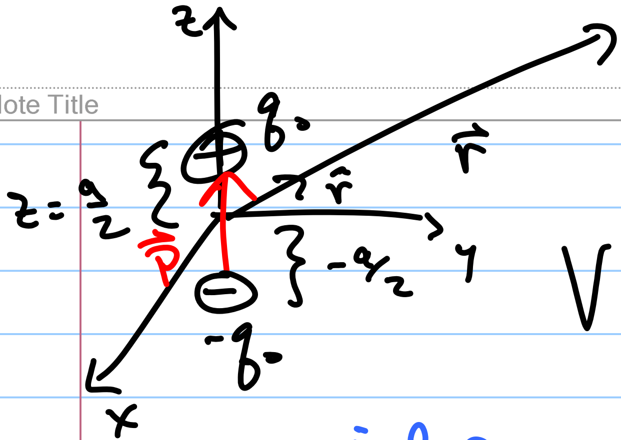


use dipole approx

$$V = V_0 + V_1 + \cdots$$

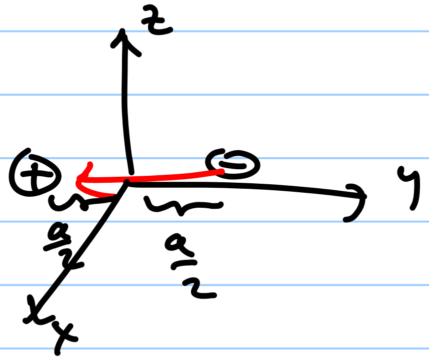
monopole dipole

coord indep

$$\boxed{V_{dipole} = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}}$$

$$\vec{p} = \sum_i q_i \vec{r}_i$$

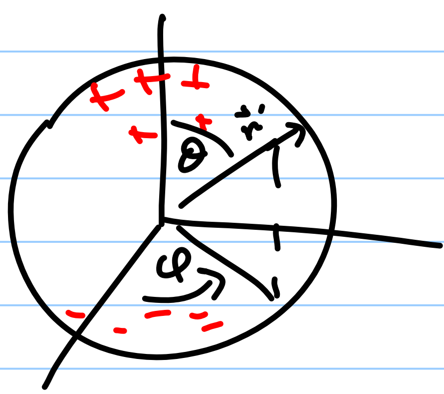
$$\vec{p} = q_0 \frac{a}{2}\hat{z} - q_0\left(-\frac{a}{2}\right)\hat{z} = q a \hat{z}$$



$$\vec{p} = q_0\left(-\frac{a}{2}\right)\hat{y} - q_0\frac{a}{2}\hat{y}$$

$$= -q_0 a \hat{y}$$

$$\vec{p} = \sum q_i \vec{r}_i \longrightarrow \int \vec{r}\,' dq = \int \vec{r}\,' \rho(\vec{r}\,') d\tau'$$



$$\sigma = k\cos\theta$$

$$\vec{p} = ?$$

$$\vec{r}\,' = R\hat{r}'$$

$$\sigma = k\cos\theta'$$

$$\rho = \sigma\,\delta(r' - R)$$

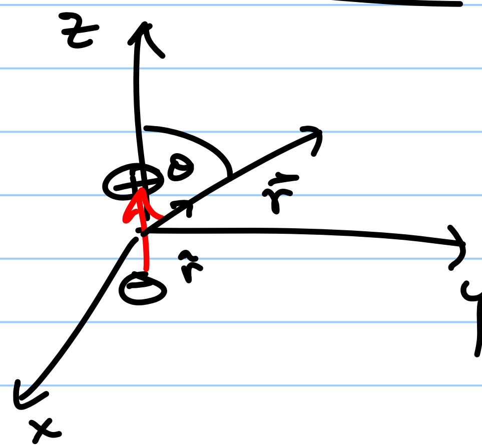
$$d\tau = r'^2 \sin\theta'\, d\theta'\, d\varphi'\, dr'$$

$$\vec{p} = \iiint_0^\infty R\,\hat{r}'\,\sigma\,\delta(r'-R)\,\underset{R}{\underset{\uparrow}{r'^2}} \sin\theta'\, d\theta'\, d\varphi'\, dr'$$

$$= \int_0^{2\pi}\int_0^{\pi} R\,\underset{\left(\sin\theta'\cos\varphi'\,\hat{x} + \sin\theta'\sin\varphi'\,\hat{y} + \cos\theta'\,\hat{z}\right)}{\underset{\parallel}{\hat{r}'}}\,\sigma\,R^2 \sin\theta'\, d\theta'\, d\varphi'$$

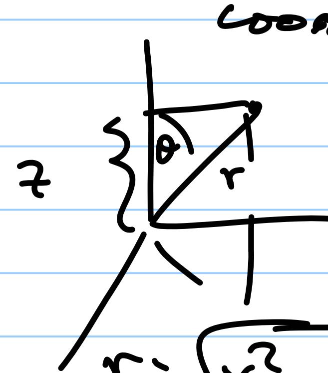
$$\int_0^{2\pi}\int_0^{\pi} R \sin\theta' \cos\varphi'\,\underset{\text{const}}{\underset{=}{\hat{x}}}\,\sigma\,R^2 \sin\theta'\, d\theta'\, d\varphi' \qquad \hat{x} \text{ part}$$

$$V = \frac{\vec{p}\cdot\hat{r}}{4\pi\epsilon_0 r^2} \quad \text{approx}$$



$\vec{E} = -\vec{\nabla} V$ find $\vec{E}$ in cartesian coords

$$V = \frac{p\,|\underset{1}{\underset{\parallel}{\hat{r}}}|\cos\theta}{4\pi\epsilon_0 r^2}$$



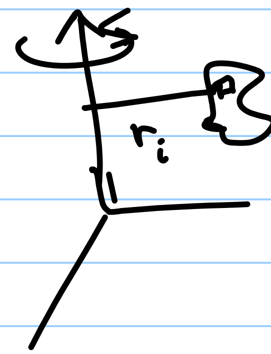
$z = r\cos\theta$

$r = \sqrt{x^2+y^2+z^2}$

$$V = \frac{p \frac{z}{r}}{4\pi\epsilon_0 r^2} = \frac{p z}{4\pi\epsilon_0 r^3} = \frac{p z}{4\pi\epsilon_0 \left(\sqrt{x^2+y^2+z^2}\right)^3}$$

$$\vec{E} = -\hat{x}\frac{\partial}{\partial x}V - \hat{y}\frac{\partial}{\partial y}V - \hat{z}\frac{\partial}{\partial z}V$$

moments



- moment of $I = \sum_i \overset{\text{mass}}{m_i}\, r_i^2$

$$\rightarrow \int r^2 \underset{\rho\, d\tau}{\underset{\shortparallel}{dm}}$$

I depends on location of axis of rotation