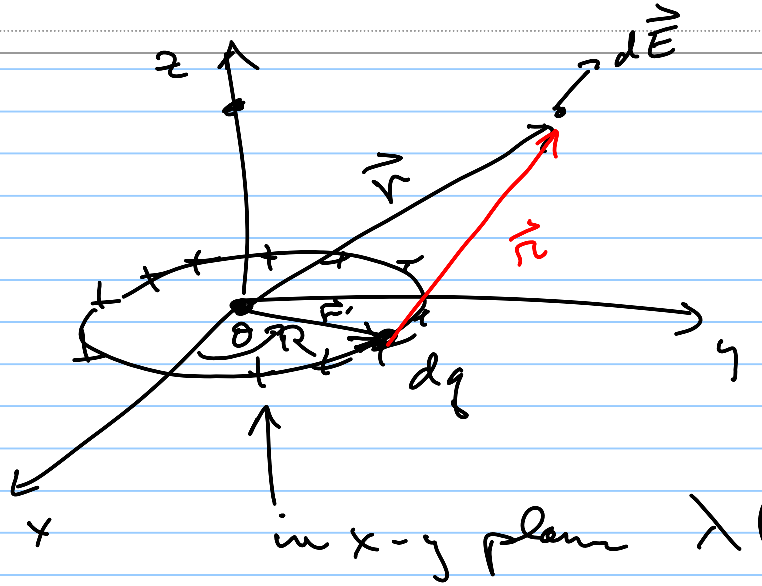
Ex II



in x-y plane λ (Coul/m) uniform on a ring

Principle: Coulomb's law + superpos

Method $d\vec{E} = k \frac{dq}{\mathscr{r}^2}\hat{\mathscr{r}}$

$dq = \lambda R d\theta'$

$\int dq = \int \lambda\, dl = 2\pi R \lambda$

$\int_0^{2\pi} dl = 2\pi R$

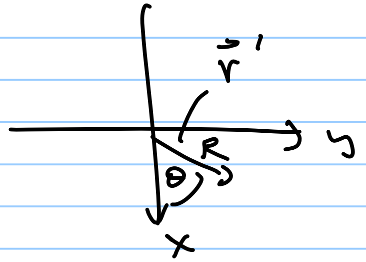
$dl = R d\theta'$

$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,'$

$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

$\vec{r}\,' = R\cos\theta'\hat{x} + R\sin\theta'\hat{y}$

$\hat{\mathscr{r}} = \frac{\vec{\mathscr{r}}}{|\mathscr{r}|}$



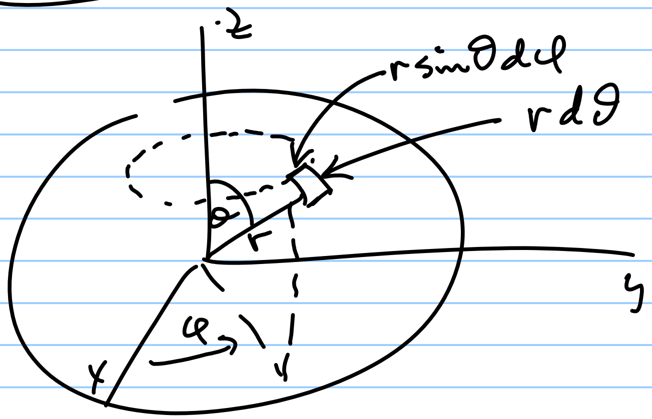
$\vec{\mathscr{r}} = (x - R\cos\theta')\hat{x} + (y - R\sin\theta')\hat{y}$
$+ (z - 0)\hat{z}$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_0^{2\pi} \frac{\lambda R\, d\theta'}{\left[(x-R\cos\theta')^2+(y-R\sin\theta')^2+z^2\right]^{3/2}} \left[(x-R\cos\theta')x+(y-R\sin\theta')\hat{y}+z\,\hat{z}\right]$$

Check :

- $|\vec{r}|$ get very large $E \rightarrow \frac{kQ}{r^2}$
- along z-axis $E \rightarrow E\,\hat{z}$
- at $x = 0 = y = z$ $\vec{E} = 0$
- $R \rightarrow 0$ get $E \rightarrow \frac{kQ}{r^2}$
- $R \rightarrow \infty$?

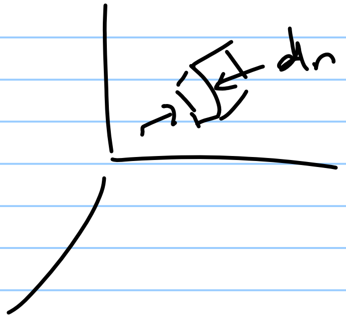
Area of sphere



area = Σ area each tile = Σ ΔA → $\int_0^{\pi}\int_0^{2\pi} r \sin\theta \, d\varphi \, r \, d\theta$

area = $4\pi r^2$

volume = $\iiint \left(\text{area tile}\right) \underbrace{\text{thickness}}_{dr}$

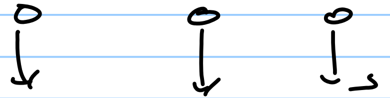


$\int_0^{R}\int_0^{\pi}\int_0^{2\pi} r \sin\theta \, d\theta \, r \, d\varphi \, dr = \frac{4}{3}\pi R^3$

θ

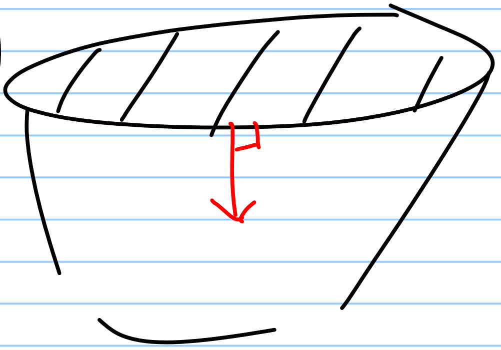
φ

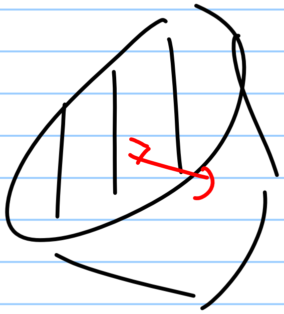
Flux:



$\vec{J}$ in direction of flow

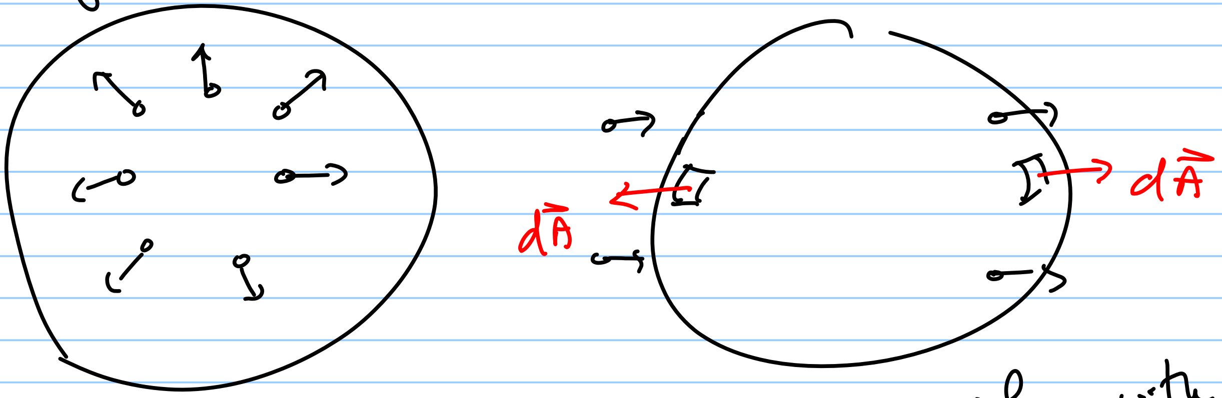
$\vec{J}$ flux $\left(\frac{\#\text{ drops}}{\text{sec area}}\right)$





$$\vec{J}\cdot\vec{A} = |\vec{J}||\vec{A}|\cos\phi \qquad |\vec{J}||\vec{A}|\cos 45$$

Conservation of raindrops



$$\Phi \quad \frac{\#\text{drops}}{s} = -\frac{d}{dt} D_{\text{enclosed}} \qquad D = \int \rho \, dVol$$

surface

drops enclosed by that surface

volume within surface

drop density

$\frac{\#\text{drops}}{\text{volume}}$

In a static situation $\frac{d}{dt} \rightarrow 0$

$$\Phi = 0$$

Add a source of raindrops (sprinkler in center)

$$\Phi = -\frac{d}{dt} D + S'$$

↑ Source of raindrops

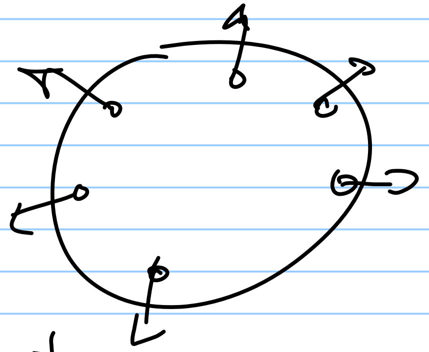
For conservation of charge $S' \equiv 0$

Static case

$$\Phi = S'$$

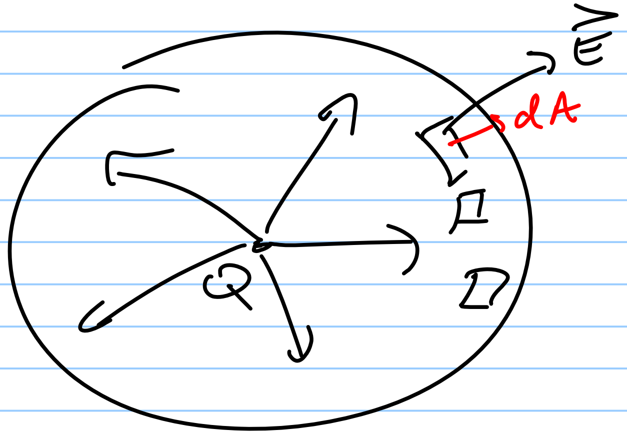
↑ $10^3 \frac{\text{drop}}{\text{sec}}$

↑ sprinkler spewing out 10^3 drops/sec



E field flux

$$\Phi = \int \vec{E} \cdot d\vec{a}$$



$$\overset{\curvearrowleft}{\int} |\vec{E}| |d\vec{a}| \cos\phi$$

$$= |\vec{E}| \int |d\vec{a}|$$

$$= E \, 4\pi r^2 = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} 4\pi r^2 = \frac{Q}{\epsilon_0} \Rightarrow \$$$